Kotani's Ant Problem

Projects for my (and maybe your) classes

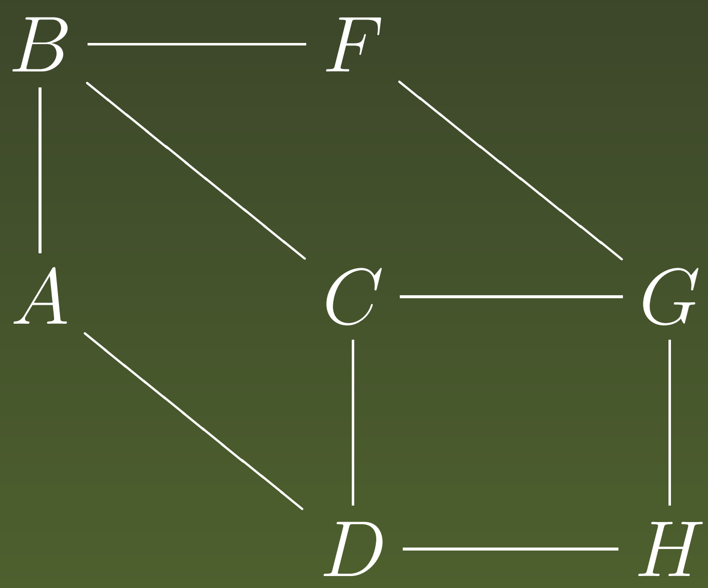
Adam E. Parker

aparker@wittenberg.edu

Wittenberg University

The Problem

Q: An ant is at corner A of a $1 \times 1 \times 2$ box. It crawls along the surface, along a geodesic, the shortest possible path, to a point B. Where is B located to make the path as long as possible?



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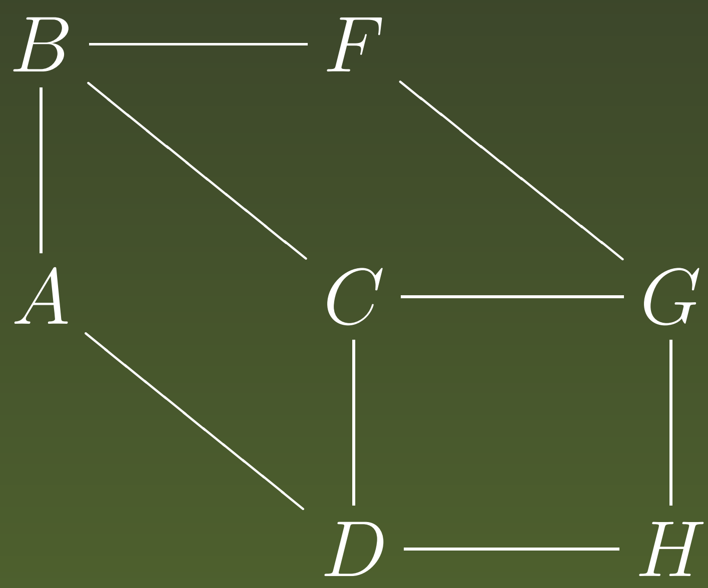

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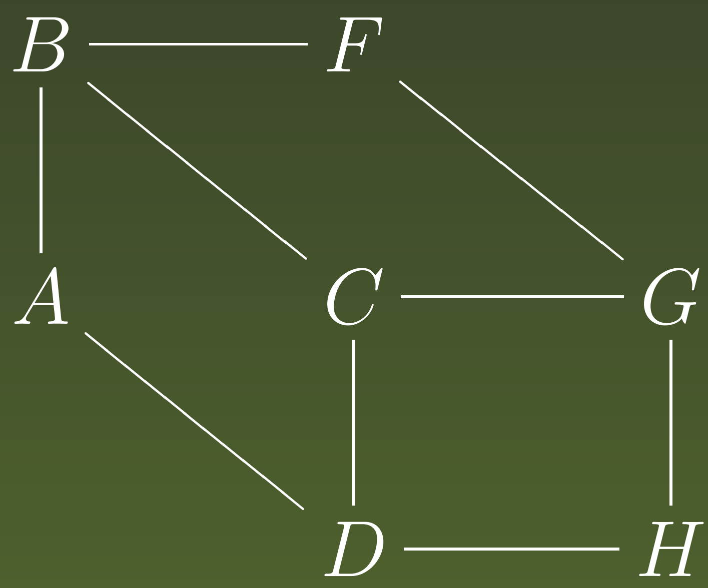
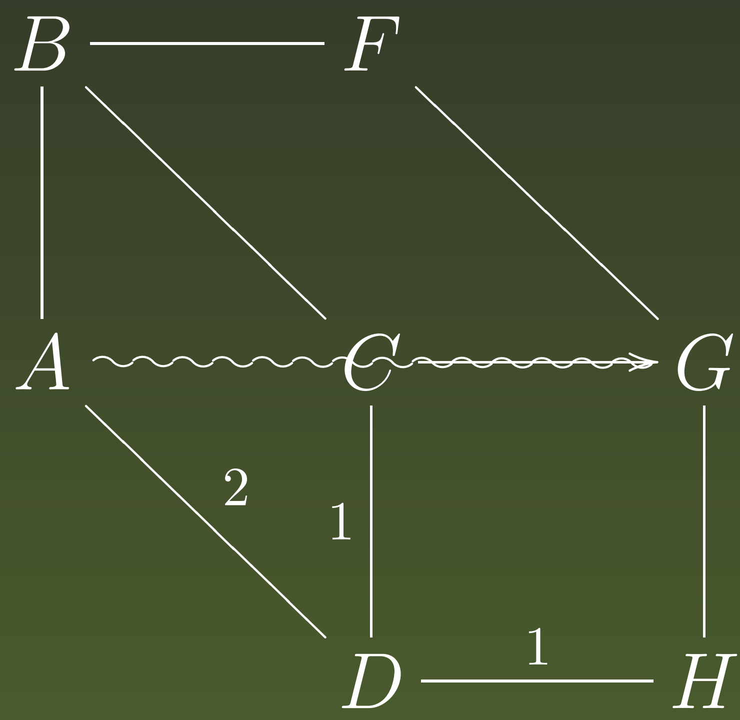


- A: We (and your students) would all guess point G.

What's the best path?

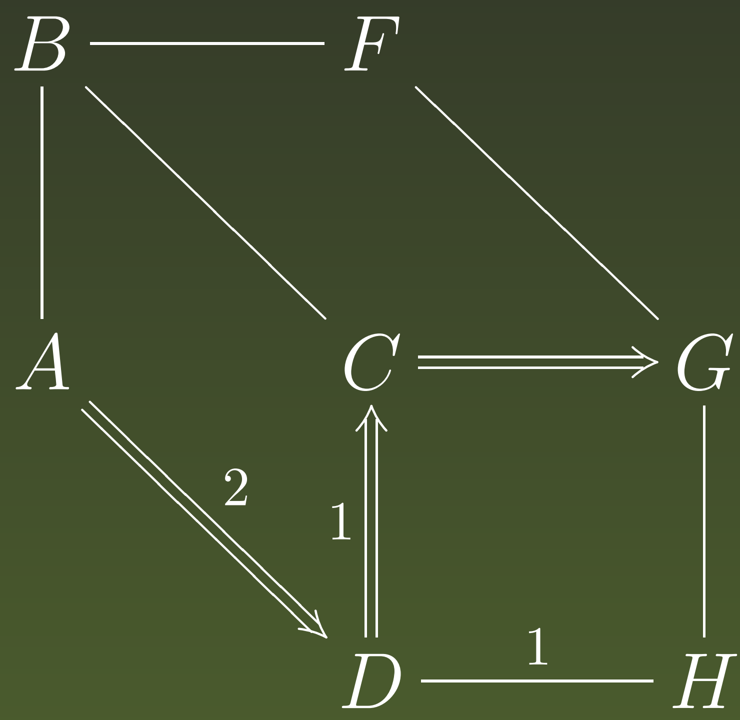
The first guess always seems to be "in" the box.



We explain that this isn't a burrowing ant.

What's the best path?

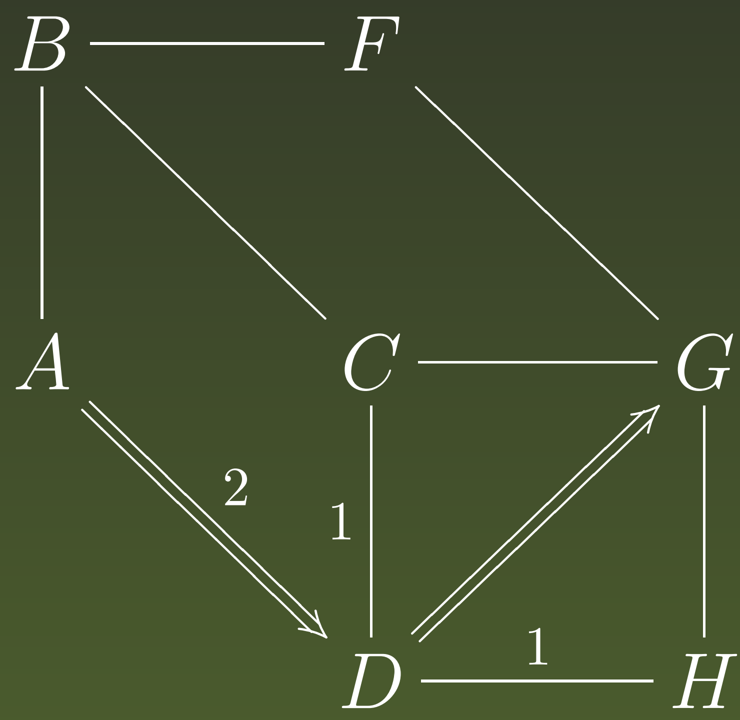
The second guess is typically not the best path.



We can easily see that this path has length 4.

What's the best path?

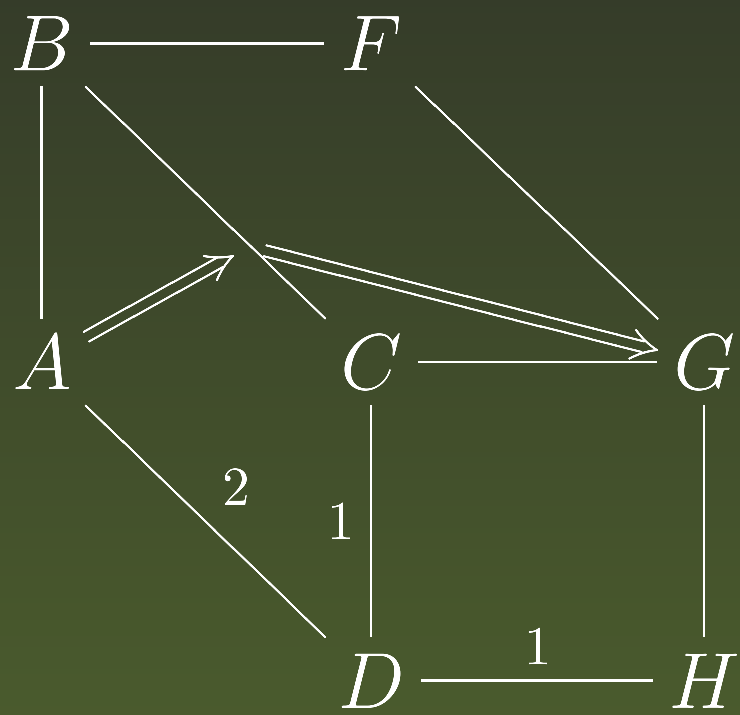
The third guess is a little better.



We can easily see that this path has length $2+\sqrt{2} = 3.414\ldots$.

What's the best path?

The fourth guess tends to be better still.

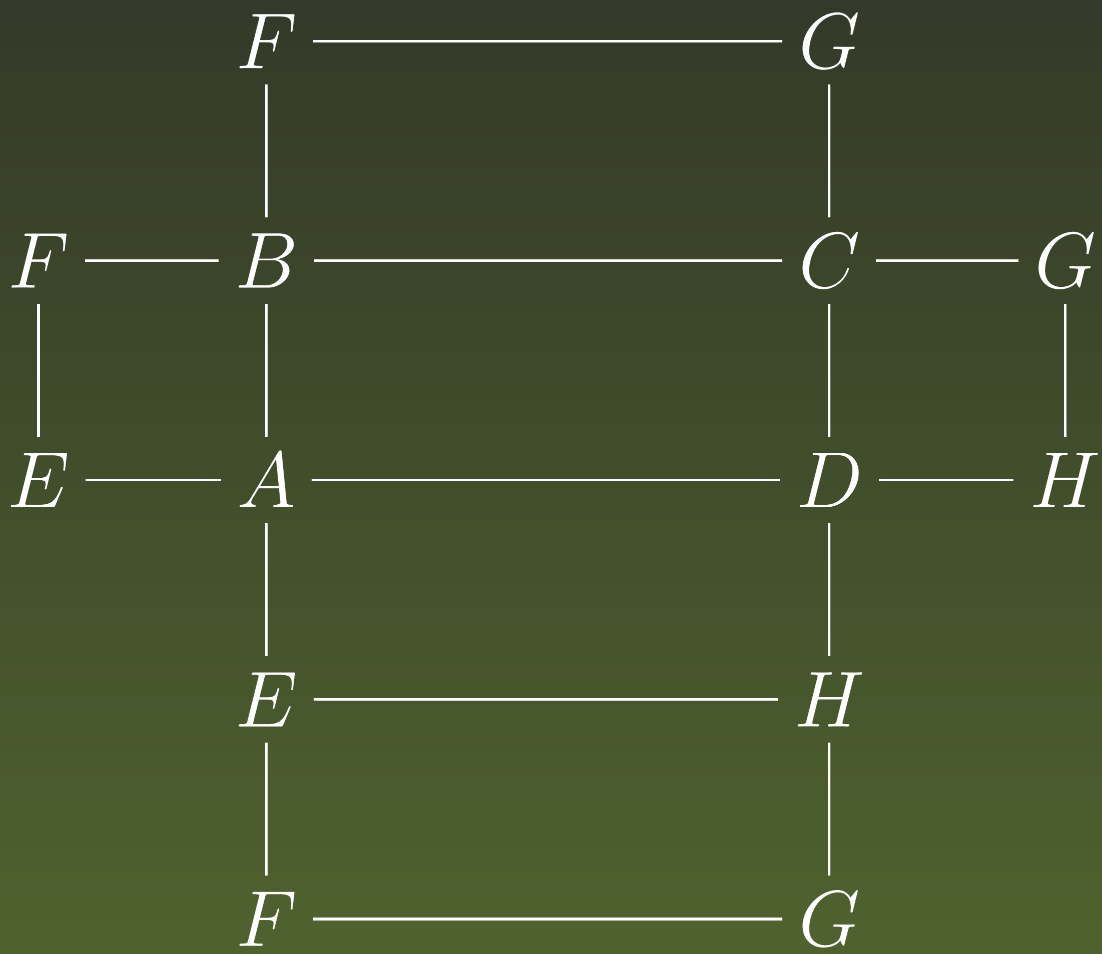


But how do we find this length?

The Best Path

Let's "open up" the solid, and ask what's the shortest path between A and G?

The Best Path



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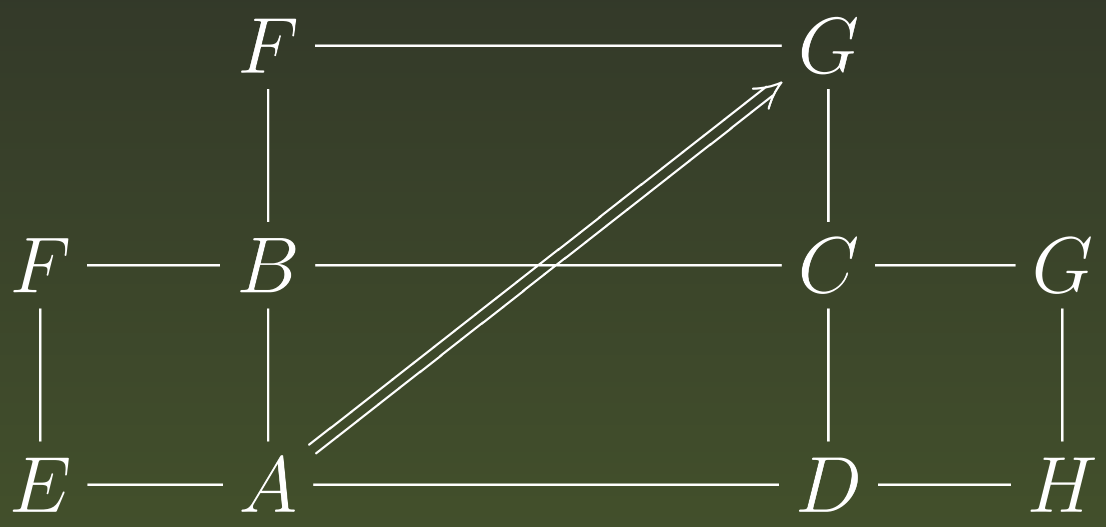

It's easy to see that this path has length $\sqrt{8}$, and it is the shortest path from A to G.

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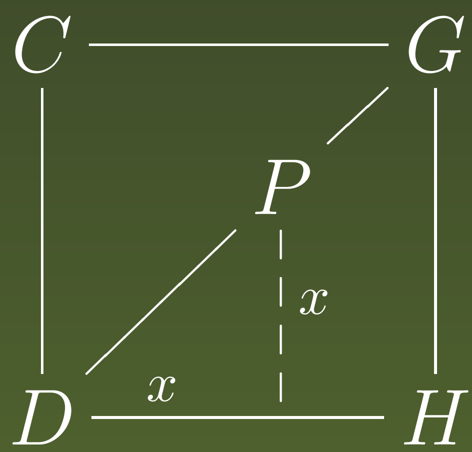

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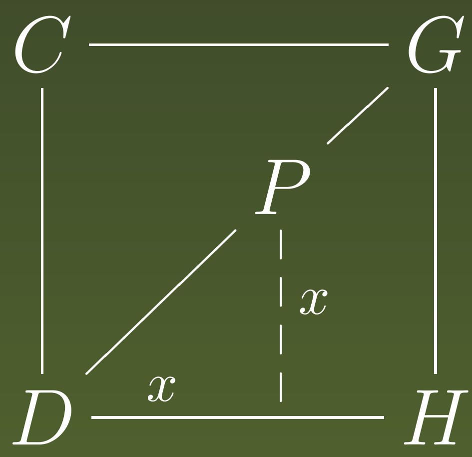
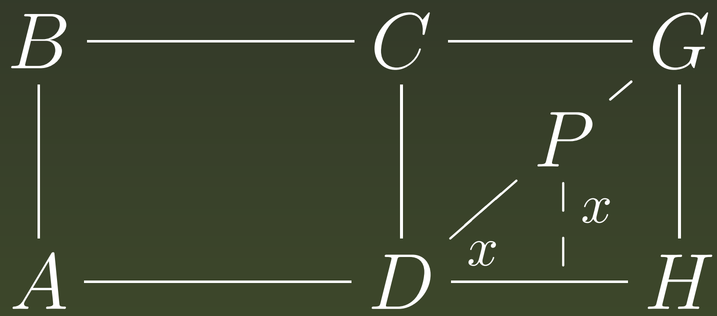


- What is the shortest distance from A to P?

First Path



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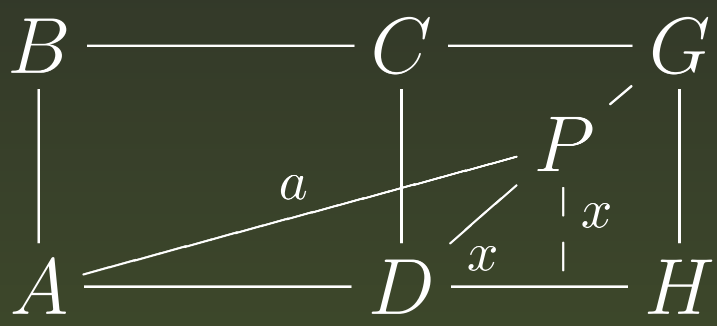

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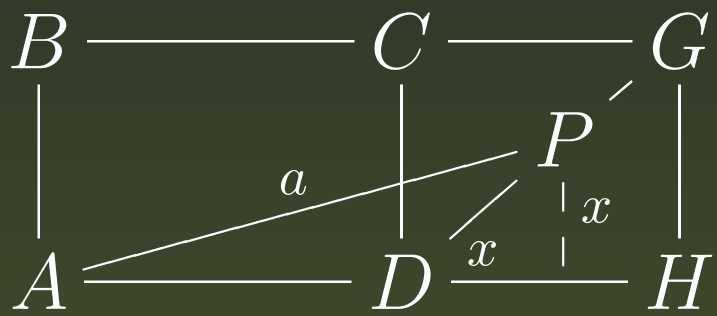

$$a^2 = (2 + x)^2 + x^2$$

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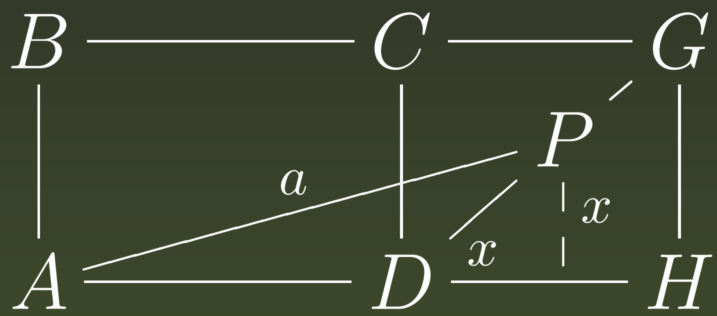

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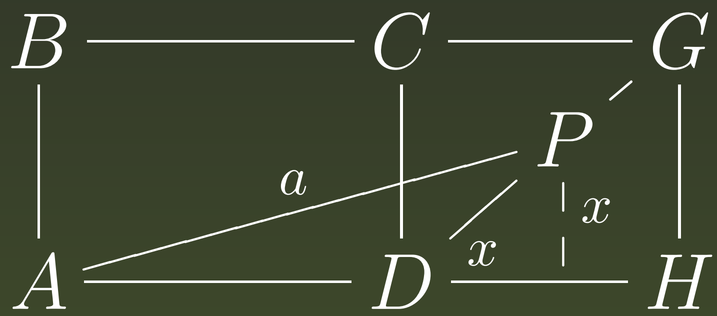

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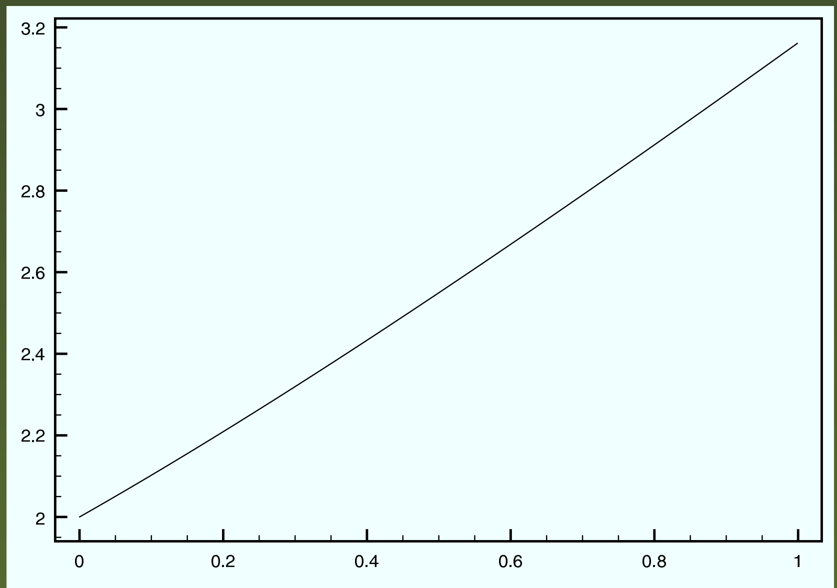

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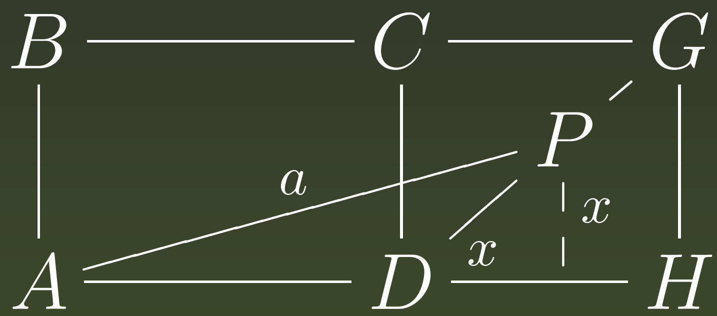

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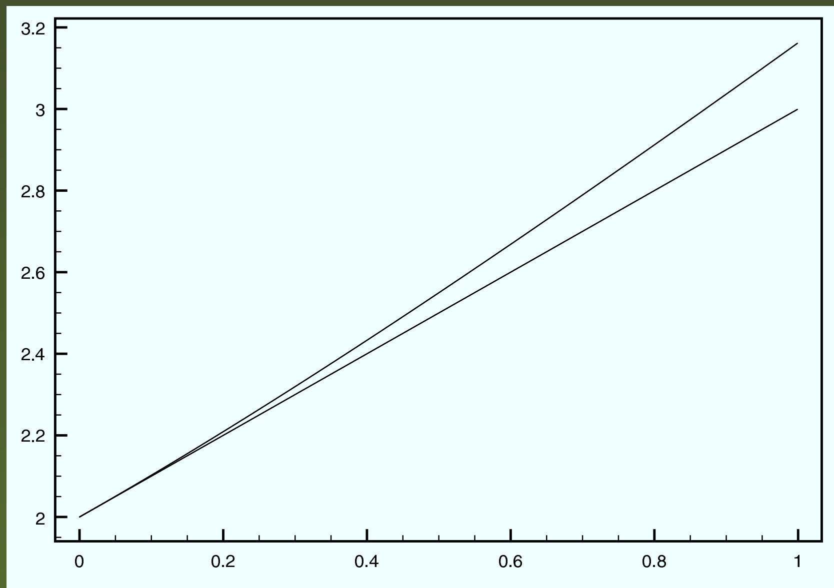

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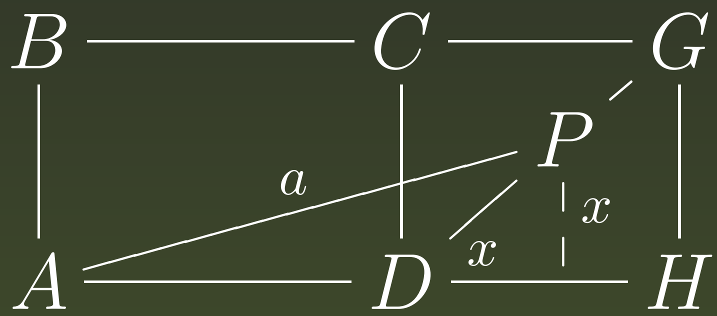

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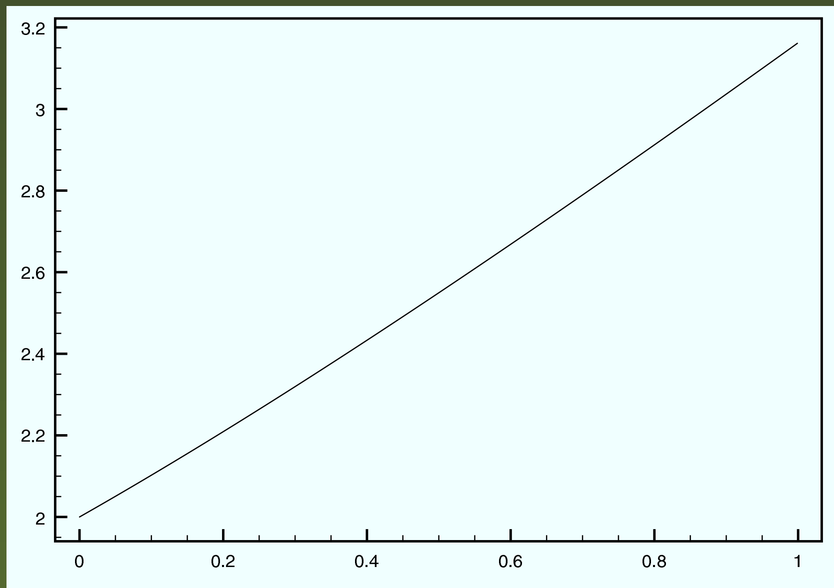

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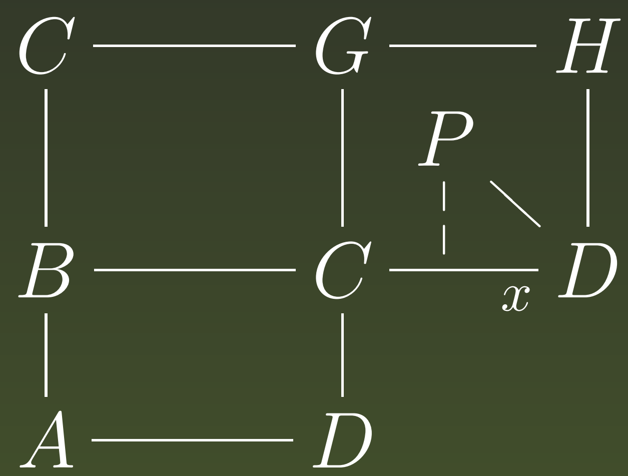

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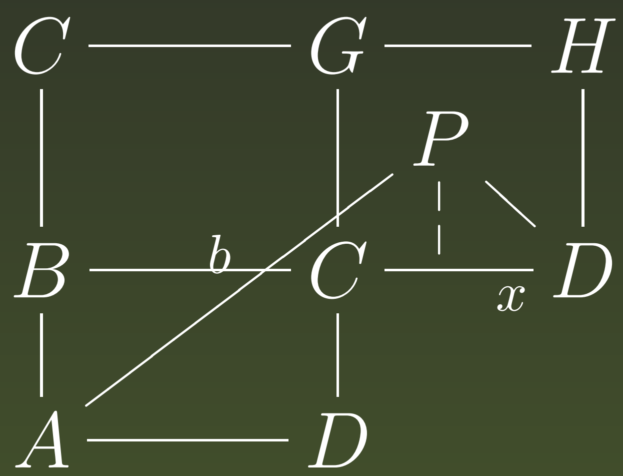

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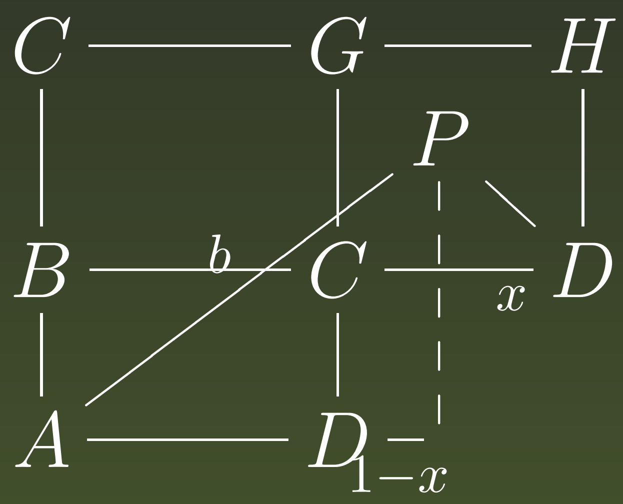

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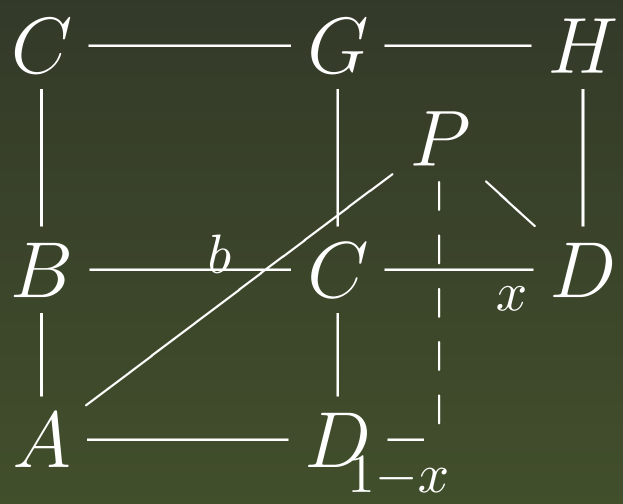

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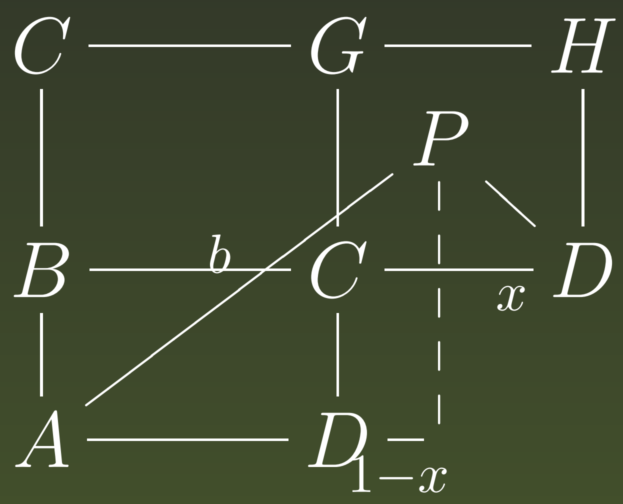

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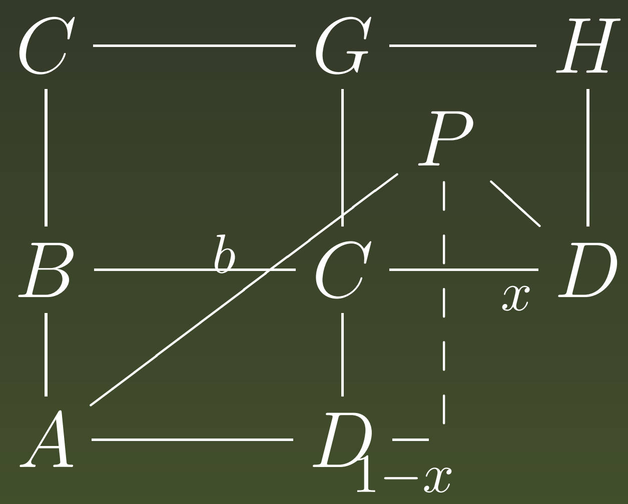

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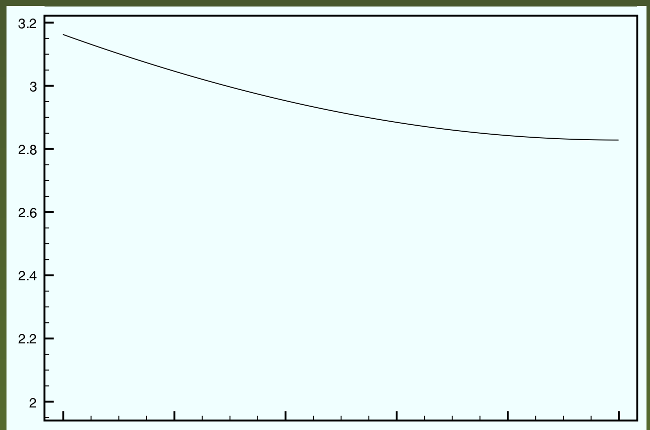
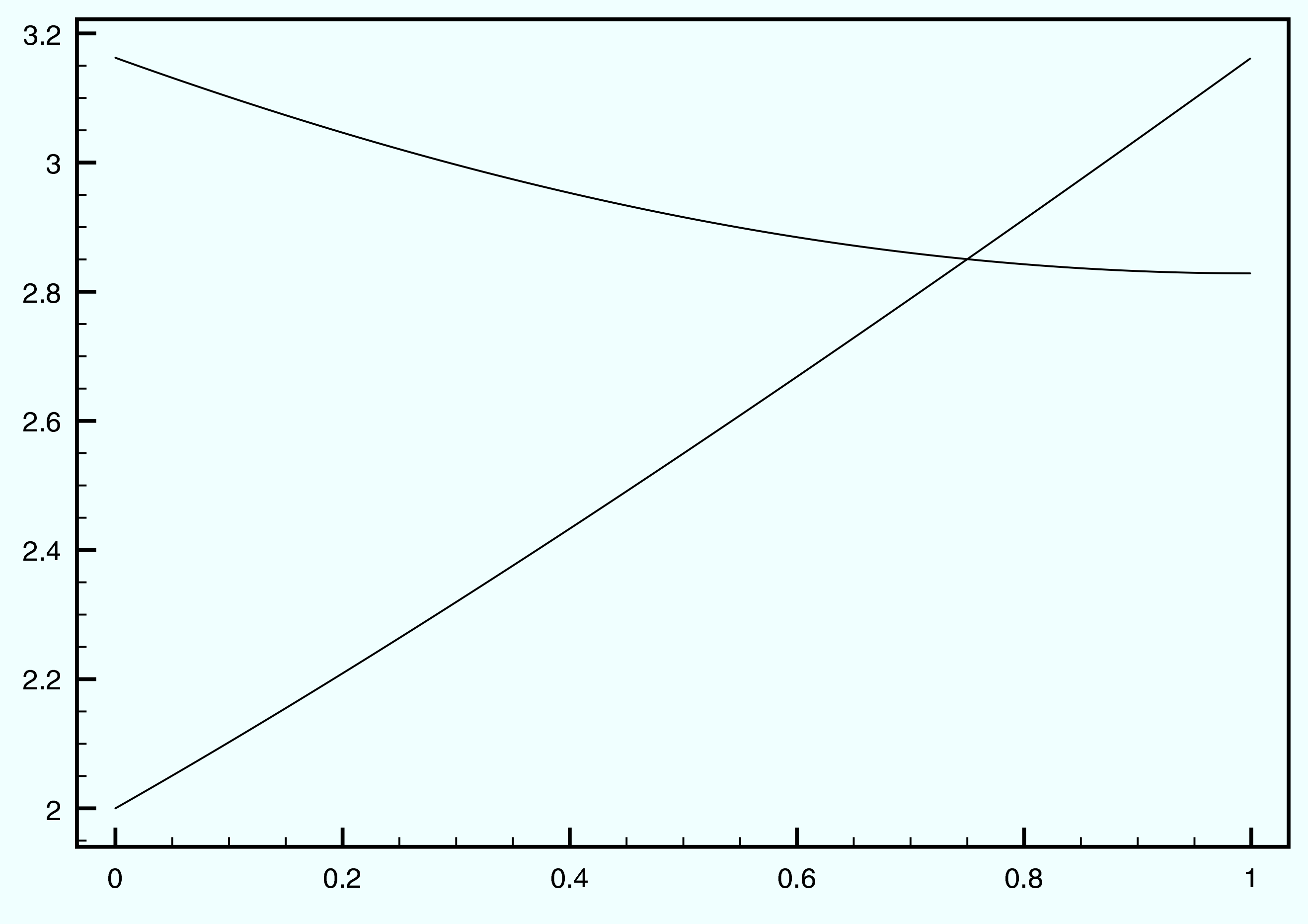


Maximum distance from A

Let’s examine the graph closer



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We've really found a farther point!

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- I ask for them to come up with other questions.

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- It's easy to make easier. For example, rather than finding the point P, you can tell the students where it is. This avoids the modeling.

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- Higher dimensional analogues?

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- You get the idea

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- References were at the beginning.

Conclusion

- I believe that this is a fun way of using the Pythagorean Theorem to model distances in a (perhaps) new an unexpected way.
- References were at the beginning.
- Thank you for listening.