Comp 260: Computational Models and Methods
Numerical Search Techniques – Ch. 7.6

Optimizing Differentiable Functions
Unimodal Functions
Search Method Paradigms
Dichotomous Search
Golden Ratio Search
Model Fitting Criterion – Minimizing $$\sum_{i=1}^{n} |y_i - c - x_i|$$

From Calculus
1. Extreme Value Theorem: A continuous function in a closed interval has an absolute maximum and an absolute minimum
2. Max-Min Theorem for Differentiable Functions: If a differentiable function $$f(x)$$ has a relative maximum or relative minimum at a point c, then $$f'(c) = 0$$
3. A differentiable function on a closed interval has an absolute maximum (absolute minimum) at a critical point or at an endpoint.

Geometric Picture of Unimodal Search Method

Dichotomous Search: Compute $$x_1 = \frac{a+b}{2}$$ and choose $$x_1, x_2 = \frac{a+b}{2} \pm \varepsilon$$ for some small $$\varepsilon > 0$$

Golden Ratio: Set $$r = 0.618$$ and set $$x_1 = a + (1 - r)(b - a)$$
$$x_2 = a + r(b - a)$$ where
$$\frac{1 - r - r}{r} \Rightarrow r = \frac{1 \pm \sqrt{5}}{2} \approx 0.618$$

Golden Ratio Search
1. Choose tolerance $$\varepsilon > 0$$
2. Set $$r = 0.618$$
3. Calculate $$f(x_1)$$ & $$f(x_2)$$
4. If $$f(x_1) \leq f(x_2)$$ - new interval is $$[x_1, b]$$
   - $$a \leftarrow x_1$$;
   - $$x_1 \leftarrow x_2$$;
   - $$x_2 = a + r(b - a)$$;
   - $$f(x_1) \leftarrow f(x_2)$$; calculate $$f(x_1)$$
5. Else – new interval is $$[a, x_2]$$
   - $$b \leftarrow x_2$$;
   - $$x_2 \leftarrow x_1$$;
   - $$x_1 = a + (1 - r)(b - a)$$;
   - $$f(x_2) \leftarrow f(x_1)$$; calculate $$f(x_2)$$
6. If $$b - a < \varepsilon$$ then stop else goto 4

Model Fitting Example