Lanchester Combat Models: Combat between two homogeneous forces

Combatant 1: \( x(t) \) – strength (tanks?) at time \( t \)

Combatant 2: \( y(t) \) – strength (tanks?) at time \( t \)

System of Differential Equations

\[
\begin{align*}
\frac{dx}{dt} &= -ay \\
\frac{dy}{dt} &= -bx
\end{align*}
\]

attrition rate coefficients

Graphical Analysis

Equilibrium Point: \( \frac{dx}{dt} = 0 = \frac{dy}{dt} \)

Rest Point: \((0,0)\)

\[
\begin{array}{c}
da \quad db \\
\frac{dx}{dt} < 0 & \quad \frac{dy}{dt} < 0
\end{array}
\]

Analytic Solution: Lanchester Square Law Model

Solving: 3 Cases: \( C < 0 \) (X wins), \( C = 0 \), \( C > 0 \) (Y wins)

If \( C > 0 \) then

\[
\begin{align*}
ay dy &= -bx dx \\
ay^2 - bx^2 &= C
\end{align*}
\]

If \( C < 0 \) then

\[
\begin{align*}
x &= \frac{\sqrt{C}}{a} \quad y &= \frac{\sqrt{C}}{-b}
\end{align*}
\]

Other Models: \( a = a(t, x/y) \) includes force ratio factor

\[
\begin{align*}
\frac{dx}{dt} &= -axy \\
\frac{dy}{dt} &= -bxy
\end{align*}
\]

Economic Aspects of Arms Race (Richardson Model)

Country X: \( x(t) \) amount spend on defense

Country Y: \( y(t) \) amount spent on defense

\[
\begin{align*}
\frac{dx}{dt} &= -ax + by + c \\
\frac{dy}{dt} &= mx - ny + p
\end{align*}
\]

\( a \): decrease at rate proportional to amount being spent

\( b \): increase at rate proportional to amount \( Y \) is spending

\( c \): underlying grievances

See LanchesterModelsSp15.nb