Write up carefully the following

1. Using the Pumping Lemma prove that the language $L = \{a^n b^k a^l \mid k \geq n + l\}$ is not regular.

2. Using the Pumping Lemma prove that the language $L = \{a^k \mid k \text{ is prime}\}$ is not regular. Hint: If $w = xyz$ for $|w| \geq m$ where $x = a^\alpha, y = a^\beta, z = a^\gamma$ and $m = \alpha + \beta + \gamma$ is prime (where $\beta \geq 1$ and $\gamma \geq 1$) then note that $\alpha + \beta(\alpha + \gamma) + \gamma = (\beta + 1)(\alpha + \gamma)$ is not.

3. Determine which of the following languages is regular and which is not. Construct a DFA for the regular language and use the Pumping Lemma to show that other is not regular.

   $L_1 = \{a^n b^n \mid n \geq 1\} \cup \{a^n b^m \mid n \geq 1, m \geq 1\}$
   
   $L_2 = \{a^n b^n \mid n \geq 1\} \cup \{a^n b^{n+2} \mid n \geq 1\}$