Introduction: In class today (08/26/2010) I handed out an outline for the proof that there is no largest prime. The purpose of this assignment is to work out the details for the proof (and in doing so understand it).

Recall the following definition of “divides”: For integers $a$ and $b$ we say $a$ divides $b$, written $a | b$, if and only if there is an integer $k$ such that $b = k \cdot a$.

Do the following:

A. Write out an argument (proof) for the following lemma which we discussed in class today.

Lemma (transitive property of divides): For integers $a$, $b$, and $c$, if $a | b$ and $b | c$ then $a | c$.

Hint: Recall that we want to show there is an integer $n$ such that $c = n \cdot a$.

B. Using what you learned in making the argument for the lemma above, write out an argument (proof) for the second lemma.

Lemma (linear combination property): For integers $a$, $b$, and $c$, if $a | b$ and $a | c$ then $a$ divides any linear combination of $b$ and $c$; that is $a | (r \cdot b + s \cdot c)$ for any integers $r$ and $s$.

Hint: What are you trying to show?

Notes:

1. The above is not intrinsically difficult to do. Think! I’m interested in seeing if you understand why the two lemmas are true and if you can communication that fact to me.
2. You may collaborate but all write up must be your own; I want to see your own words, thoughts and expressions. Feel free to ask or e-mail questions if you have difficulty.
3. Do NOT use outside sources (e.g. on-line sources) for the answer. Work with each other and try to figure it out for yourselves.
4. Most word processors (e.g. MS-Word) have minimal equation editing capabilities so use them. Any mathematical notation you’ll need for this assignment is minimal.
5. Grading will be on neatness, clarity and completeness of presentation and of course correctness!
6. Please hand in this assignment sheet with your assignment!