Introduction: In class today (09/02/2010) we completed the proof that there is no largest prime. The purpose of this assignment is to complete the details for the proof (and in doing so understand it).

For completeness begin by redoing Part 1 (A and B below - handed in today) adding any improvements noted in class

A. Write out an argument (proof) for the following lemma.

Lemma (transitive property of divides): For integers $a$, $b$, and $c$, if $a|b$ and $b|c$ then $a|c$.

Hint: Recall that we want to show there is an integer $n$ such that $c = n \cdot a$

B. Write out an argument (proof) for the second lemma.

Lemma (linear combination property): For integers $a$, $b$, and $c$, if $a|b$ and $a|c$ then $a$ divides any linear combination of $b$ and $c$; that is $a| (r \cdot b + s \cdot c)$ for any integers $r$ and $s$. Hint: What are you trying to show?

Recall the following definition of “divides”: For integers $a$ and $b$ we say $a$ divides $b$, written $a|b$ if and only if there is an integer $k$ such that $b = k \cdot a$

Definition: An integer $p > 1$ is prime if and only if its only divisors are 1 and itself.

Definition: An integer $c > 1$ is composite if and only if it is not prime; i.e. there is an integer $k$ greater than 1 and less than $c$ such that $k|c$

C. Write out the argument (proof) for the following lemma.

Lemma: Every integer is either prime or composite

Well-Ordered Property: Every non-empty subset of positive integer has a least element

D. Write out the argument (proof) for the following lemma.

Lemma: Every positive integer $n > 1$ has a prime divisor

Hint: Use the Well-Ordered Property

E. Write out an argument (proof) for the following theorem.

Theorem: There is no largest prime (i.e. IX.20 “prime numbers are more than any assigned multitude of primes”)

Hint: Assume false and derive a contradiction!