Math 120: Elementary Functions

Today’s Overview

Linear Models
Do you know these geometric formulas?
Finding Linear Models from Data
Scatter Plot and Least Squares Regression Lines

Equations of Lines – A Review

Slope: \[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

Point Slope Form
\[ y - y_1 = m(x - x_1) \]

Slope Intercept Form
\[ y = mx + b \]

Given two points \((x_1, y_1)\) and \((x_2, y_2)\), how would you find the equation of the line?

1. The Welcome Home apartment rental company has 1600 units available of which 800 are currently rented at $300 per month. A market survey indicates that each $5 decrease in monthly rent will result in 20 new leases.

Find a linear function that expresses units rented as a function of price.

2. The Sweet Drip Beverage Company sells cans of soda in machines. It finds that sales average 26,000 can per month when the cans sell for $0.50 each. For each nickel increase in price, the sales drop by 1000 cans.

Assuming a linear relationship between cans sold and price per can, express number of cans sold as a function of price.

3. The 30 sales associates at Athens Wire Company average $50,000.00 in sales per month. If each additional hire reduces the average sales by $1000.00 per month find the linear function that gives average sales as a function of the number of sales associates.

Do You Know These Geometric Formulas?

1. Area of Triangle
2. Area of Trapezoid
3. Volume of a Rectangular Solid
4. Surface Area of a Rectangular Solid
5. Circumference of a Circle
6. Area of Circle
7. Volume of Cylinder
8. Surface Area of Cylinder
9. Volume of Cone
10. Volume of Sphere
11. Surface Area of Sphere

Example 4 – Volume of a Box

A square of side x inches is cut out of each corner of an 8 by 15 inch piece of cardboard and the sides are folded up to form an open top box.

a. Write volume V of the box as a function of x
b. What is the domain of V(x)?
c. Graph V(x) over its domain and using the maximum finder to find the maximum.
d. What is x?

Example 5: Table P.3 US Imports From Mexico

<table>
<thead>
<tr>
<th>Year</th>
<th>Billions of Dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>135.9</td>
</tr>
<tr>
<td>2001</td>
<td>131.3</td>
</tr>
<tr>
<td>2002</td>
<td>134.6</td>
</tr>
<tr>
<td>2003</td>
<td>138.1</td>
</tr>
<tr>
<td>2004</td>
<td>155.9</td>
</tr>
<tr>
<td>2005</td>
<td>170.1</td>
</tr>
<tr>
<td>2006</td>
<td>188.2</td>
</tr>
<tr>
<td>2007</td>
<td>210.7</td>
</tr>
</tbody>
</table>

1. Create a scatter plot
2. Choose two points that “best” approximates a linear model for the data; find the linear equation
Example 5 - Doing Scatter Plots

- [STAT] Edit
  - 1:Edit [Enter] Enter (x,y) coordinates in L1 and L2
- [2nd] [STAT PLOT]
  - 1:Plot1 On Off [Enter]
  - Type
  - Xlist: L1
  - Ylist: L2
  - Mark
- [WINDOW] use data to set Xmin, Xmax, Ymin, Ymax, etc
- [GRAPH] Important! Make sure no functions are active

Example 2 cont.

Finding a linear model which “best” fits the data

Definition: A residual is \( \frac{\text{observed} - \text{computed}}{\text{computed}} \)
measures the error between data and model.

Given a set of data points \((x_i, y_i)\), \((x_2, y_2)\),...,(\(x_n, y_n\)) the linear regression line is the unique line \(y = ax + b\) that minimizes the sum of the squared residuals. A residual is the vertical (or y) distance between \(y_i\) and \(ax_i + b\) for \(i = 1, 2,..,n\); that is, determine the parameters \(a\) and \(b\) that minimizes

\[
\sum_{i=1}^{n} \left(y_i - (ax_i + b)\right)^2
\]

Example 6 – (# 50 p. 170)

1. Generate a scatter plot
2. Pick two data points and use them to compute a linear model.
3. Find & graph the least squares regression line
   - [STAT] Calc 4:LinReg (ax+b)
   - [VARS] 5:Statistics [ENTER] EQ 1:RegEQ

Example 7 – (#68, p. 172)

1. Generate a scatter plot
2. Pick two data points and use them to calculate a linear model.
3. Find & graph the least squares regression line
   - [STAT] Calc 4:LinReg (ax+b)
   - [VARS] 5:Statistics [ENTER] EQ 1:RegEQ

Written Homework #13 – Due W 10/2/13

1. – 3. Write up the 3 linear equation examples from class
4. Do (or redo) Example 4 from class
5. Redo Example 4 but this time assume you have a 12 inch by 12 inch square sheet of paper.
6. Do Example 5 from class. Use years 2001 and 2004 for your two data points for your best estimate linear model. Compare with the least squares regression line.
7. Do the same for Example 6 from class. Use data points for ages 10 and 60 for your best estimate.
8. Do the same for Example 7 from class. Use data point for the years 1960 and 2000 for your best estimate.

For questions 6 – 8 be sure to write down the equations of both lines. Graph other both on top of scatter plot will let you see how close they are to the data.