Math 120: Elementary Functions
Ch. 2.8 – Solving Inequalities (Using Sign Charts)

- Polynomials – Factor First
- Rational Functions – Factor Numerator & Denominator
- Characterizing Vertical Asymptotes
- Absolute Value
- Square Roots – Check Domains

Solving Inequalities – Using Sign Charts

Solve \( x^2 - 4x - 21 > 0 \)

Do the following:
1. Find the zeros (factor first)
2. Plot the zeros on a number line for each factor
3. Test the sign of the factor on each sub-interval
4. Combine the factor sign lines
5. Answer the question
6. Grapher check?

Rational Inequalities

Solve \( \frac{x^2 - 9}{x^2 - 3x - 28} < 0 \)

Do the following:
1. Factor numerator and denominator and find the zeros
2. Plot the zeros on a number line
3. Test the sign on each sub-interval
4. Combine the factor sign lines
5. Answer the question

Characterizing Vertical Asymptotes

If you know the sign of a rational function to the left and right of a vertical asymptote, you can characterize the asymptotes.

\[
f(x) = \frac{x^2 - 9}{x^2 - 3x - 28}
\]

Use the sign line to characterize the vertical asymptotes; i.e. evaluate

\[
LHL \lim_{x \to -7} f(x) = \quad RHL \lim_{x \to +3} f(x) =
\]

More Exercises

1. Solve \( x^2 - 4x^2 + x + 6 \leq 0 \)
   Hint: Using synthetic division show 3 is a zero then go on and find the other zeros to obtain a complete factorization.

2. Solve \( \frac{2x^2 - 5x - 7}{x + 5} < 0 \)

More Examples – Same Technique

1. Solve \( f(x) = \frac{x - 3}{\sqrt{x + 1}} > 0 \) Domain?

2. Solve \( f(x) = \frac{x + 1}{x + 3} < 0 \)

3. If an \( x \) by \( x \) inch squares are cut from the four corners of a 20 inch square piece of paper and the resulting flaps bent upward to make an open top box, how big must the corners pieces be so that the volume of the box is at least 500 cubic inches?

Solve with or without a grapher – see next slide
Solution

\[ V = (20 - 2x)^2 x = 4x^3 - 80x^2 + 400x \]

Solve \( 4x^3 - 80x^2 + 400x \geq 0 \) or \( 4x^3 - 80x^2 + 400x - 500 \geq 0 \)

To find the zeroes, first factor out the 4

\[ 4x^3 - 80x^2 + 400x - 500 = 4(x^3 - 20x^2 + 100x - 125) \]

By RZT: possible rational zeros are ±1, ±25, ±125

Try 5!

\[
\begin{array}{c|ccc}
5 & 1 & -20 & 100 & -125 \\
0 & 5 & -75 & 125 & \\
1 & -15 & 25 & 0 &
\end{array}
\]

Therefore \( x^3 - 20x^2 + 100x - 125 = (x - 5)(x^2 - 15x + 25) \)

\[ Zeros \ are \ \frac{15 \pm \sqrt{25}}{2} \]

Written Homework #21 – Due W 10/30/13

From page 242

7, 9, 11, 33, 37, 39, 41, 43, 49, 59, 60 (Answer 1.6849702 < x < 9.1020392), 61

Hint on #60: Circumference \( C = 8\pi - x = 2\pi r \) and so radius \( (8\pi - x)/2\pi \). In addition \( h = \sqrt{16 - r^2} \) and the volume of a cone \( V = (1/3)\pi r^2 h \). Find volume \( V \) as a function of \( x \) and using your grapher find the intersection with \( y = 21 \).