Math 120: Elementary Functions
Today's Overview

Modeling Constant Rate of Growth (Rate of Decay)
Exponential functions:  \( y = a \cdot b^x \) (note where the variable \( x \) is)
Two cases:  \( b > 1 \) and  \( 0 < b < 1 \)

Modeling San Jose's Population
Using your grapher to solve for \( t \)
The exponential function  \( y = e^x \)
\[
\lim_{n \to \infty} \left( 1 + \frac{1}{n} \right)^n = 2.718281828459...
\]
Skip The Logistic Function (p. 258 –259)

Modeling Constant Rate of Growth
The population of a city grows at an annual rate of 5%. If the initial population is 10,000, find an equation that models the population as a function of time \( t \) in years

\[
\begin{array}{c|c}
\text{Time } t & \text{Population } P \\
\hline
0 & 10,000 \\
1 & 10,000 + 0.05 \times 10,000 = 10,500 \\
2 & 10,500 + 0.05 \times 10,500 = 11,025 \\
3 & 11,025 + 0.05 \times 11,025 = 11,576 \\
\end{array}
\]
Do you see a pattern?

Modeling Constant Rate of Decay
The half life of a radioactive substance is 22 days. If the initial amount is 100 grams find an equation that models the amount remaining as a function of time in days

\[
\begin{array}{c|c}
\text{Time } t & \text{Amount} \\
\hline
0 & 100 \\
22 & 50 = 100 \times \left(\frac{1}{2}\right) \\
44 & 25 = 100 \times \left(\frac{1}{2}\right)^2 \\
66 & 12.5 = 100 \times \left(\frac{1}{2}\right)^3 \\
\end{array}
\]
Do you see a pattern?

Modeling Constant Rate of Growth/Decay
The population of a city grows at an annual rate of \( r \%). If the initial population is \( P_0 \), find an exponential equation that models the population as a function of time \( t \) in years
Answer:  \( P = P_0 \left(1 + \frac{r}{100}\right)^t \)
The half life of a radioactive substance is \( h \) days. If the initial amount is \( Q_0 \) grams find an equation that models the amount remaining as a function of time in days
Answer:  \( Q = Q_0 \left(\frac{1}{2}\right)^{\frac{t}{h}} \)

You invest \( PV \) (present value) dollars at \( r \% \) interest compounded annually. Find an exponential equation that models the amount of money as a function time \( t \) in years.
Answer:  \( FV = PV \left(1 + \frac{r}{100}\right)^t \)

Exponential Functions
What is the difference between

\[
y = k \cdot x^a \quad \text{and} \quad y = a \cdot b^x
\]
An exponential function in \( x \) is a function that can be written in the form

\[
f(x) = a \cdot b^x
\]
where \( a \) is non-zero and \( b > 0 \) and \( b \neq 1 \). The constant \( a \) is called the initial value and \( b \) is the base

Graphing \( y = b^x \)
Two cases:  \( b > 1 \) (growth) and  \( 0 < b < 1 \) (decay)
Discuss: domain, range, continuity, \( x \)-intercepts, \( y \)-intercepts, symmetry, asymptotes, end-behavior, interval of increase/decrease, maximums/minimums, other?

Transformations:  \( y = a \cdot b^x \) (vertical stretch)
\( y = b^x \) (\( y \)-axis reflection)

Exponential functions model variables whose growth (or decay) rate is a constant – for example population growth or compound interest
Graphs of $y = b^x$

Note the 3 “Landmark” Points

If $b$ is positive and not equal to 1, there is a unique function called the exponential function with base $b$ defined by

$$f(x) = b^x \text{ for } b > 0 \text{ and } b \neq 1$$

Properties of Exponential Functions – the exponential function
1. Is defined, continuous and positive for all $x$ (domain)
2. The $x$-axis is a horizontal asymptote (no vertical asymptotes)
3. The $y$-intercept is $(0, 1)$; there is no $x$-intercept
4. If $b > 1$, $\lim_{x \to -\infty} b^x = 0$ and $\lim_{x \to +\infty} b^x = +\infty$ and $\lim_{x \to 1} b^x = 0$
   end behavior
   if $b < 1$, $\lim_{x \to +\infty} b^x = 0$ and $\lim_{x \to -\infty} b^x = +\infty$
5. If $b > 1$, $y = b^x$ is increasing
   if $b < 1$, $y = b^x$ is decreasing

Modeling San Jose’s Population
Using the table giving San Jose’s population for 2000 and 2007
find a exponential model and determine when the population will hit 1 million

Let $P(t)$ be the population $t$ years from 2000.

The Natural Base $e$
Using your calculator evaluate the expression

$$\left(1 + \frac{1}{n}\right)^n$$

For $n = 1, 10, 100, 1000, 10000, \ldots$. What do you observe?
The natural base $e$ is the limit

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

whose value is approximately $e = 2.718281828459\ldots$

$y = e^x$

is the natural exponential function!

Written Homework #22 – Due F 11/01/13

From page 261
7 – 10 (counts as 1)
15, 17, 19,
25 – 30 (part a’s only – counts as 1)
51, 53, 57, 58 (Ans: 20 gms, 5.6468 gms, 5700 years)