Math 120: Elementary Functions

Today's Overview

Extending Trig Functions Beyond Right Triangles
Alternate definitions of Trig functions
Counter-clockwise (+) and clockwise (-) rotations
Co-terminal Angles (multiples of ±2π)
Trig functions in all Four Quadrants
Find the Quadrant!
Signs of the Trig Functions: All Silver Tea Cups
Reference Angles and Reference Triangles
Quadrantal Angles
Circular Functions: the final extension: trig functions of real numbers

.... which can be extended to angles > π/2

Angle θ in standard position using counter-clockwise rotation for positive angles; clockwise for negative

\[ r = \sqrt{x^2 + y^2} \]

\[ \sin(\theta) = \frac{\text{opp}}{\text{hyp}} = \frac{y}{r} \]
\[ \cos(\theta) = \frac{\text{adj}}{\text{hyp}} = \frac{x}{r} \]
\[ \tan(\theta) = \frac{\text{opp}}{\text{adj}} = \frac{y}{x} \]

.... along with alternate/equivalent definitions of trig functions ...
Co-terminal angles differ by ±k×2π for integer k.

.... to all four quadrants.

\[
\begin{array}{c|c|c|c}
\hline
& Sin & All & Cos \\
\hline
(x < 0, y > 0) & & & \\
\hline
(x, y) & (x, y) & (x, y) & (x, y) \\
\hline
\hline
\end{array}
\]

\[ \sin(\theta) = \text{opp} \]
\[ \cos(\theta) = \text{adj} \]
\[ \tan(\theta) = \text{opp} \]

Note the signs of the trig functions in the four quadrants! (All Silver Tea Cups – where pos.)

 Extending Trig Functions Beyond Right Triangles

Place angle \( \theta \) in standard position

\[ \sin(\theta) = \frac{\text{opp}}{\text{hyp}} = \frac{y}{r} \]
\[ \cos(\theta) = \frac{\text{adj}}{\text{hyp}} = \frac{x}{r} \]
\[ \tan(\theta) = \frac{\text{opp}}{\text{adj}} = \frac{y}{x} \]

Alternate/equivalent definitions of trig functions

Find the Quadrant: Locating Angles in the 4 Quadrants

1. Counting by fractions of π: \( \pi/6, \pi/4, \pi/3, \pi/2 \)

Where is \( \theta \)?

2. Draw the reference triangle and indicate the reference angle

Reference Angles; Reference Triangles

Dropping a perpendicular from the terminal point of the ray to the x-axis forms the opposite side of the reference triangle; the reference angle \( \theta' \) is the acute angle between the ray and x-axis.
Quadrantal Angles

Angles along the x or y axis

\[
\begin{align*}
\sin(\theta) &= \frac{y}{r} \\
\cos(\theta) &= \frac{x}{r} \\
\tan(\theta) &= \frac{y}{x}
\end{align*}
\]

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>0</th>
<th>( \pi/2 )</th>
<th>( \pi )</th>
<th>( 3\pi/2 )</th>
<th>2( \pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin(\theta) )</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>( \cos(\theta) )</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( \tan(\theta) )</td>
<td>0</td>
<td>undefined</td>
<td>0</td>
<td>undefined</td>
<td>0</td>
</tr>
</tbody>
</table>

Things you ought to be able to do

1. Given the value of one trig function and the sign of a second, locate the angle (quadrant) and find the values of the other trig functions
2. Given any angle which is a multiple of \( \pi/6 \) or \( \pi/4 \) use reference angles and reference triangles to evaluate all six trig functions
3. Given the coordinates of the terminal point of the ray that determines the angle, find the values of all 6 trig functions
4. Be able to recognize 45-45-90 and 30-60-90 reference triangles to obtain the angle.

Circular Functions

Add a unit circle - so \( \theta \) is equal to the arc length \( s \)

Since \( r = \sqrt{x^2 + y^2} = 1 \)

\[
\begin{align*}
\sin(\theta) &= \sin(s) = y \\
\cos(\theta) &= \cos(s) = x \\
\tan(\theta) &= \tan(s) = \frac{y}{x}
\end{align*}
\]

Thus trig functions are defined for lengths, quantities, or any real number \( s \). So the trig function are now functions of a real variable (and not just angles).

Circular Functions - with a change of variable

Add a unit circle - so \( \theta \) is equal to the arc length \( x \)

Since \( r = \sqrt{u^2 + v^2} = 1 \)

\[
\begin{align*}
\sin(\theta) &= \sin(x) = v \\
\cos(\theta) &= \cos(x) = u \\
\tan(\theta) &= \tan(x) = \frac{v}{u}
\end{align*}
\]

Thus trig functions are defined for lengths or quantities in the variable \( x \).

Written Homework #28 - Due M 11/18/13

(17 Problems)

From page 335
41, 43, 45, 47
49, 51, 53
61, 65

From page 347
3, 5, 7, 9, 11
37, 39, 41