Math 120: Elementary Functions

Today’s Overview

Things you ought to be able to do

Quadrantal Angles
Circular Functions: the final extension: trig functions of real numbers
Revisiting the 45-45-90 and 30-60-90 Triangles
Extending the definitions of Trig functions to real numbers
Graphing the function $y = \sin(x)$
Graphing the function $y = \cos(x)$

Example 1: If $\sin(\theta) = \frac{2}{3}$ and $\cos(\theta) < 0$, locate the angle in the proper quadrant and find the values of the other trig functions.

Example 2: Sketch the reference triangle, indicate the reference angle and determine the values of the trig functions for $\theta = \frac{11\pi}{6}$.

Example 3: If point $P = (-3, -2)$ is on the terminal side of angle $\theta$, sketch the reference triangle and give the values for all 6 trig functions.

Example 4: If point $P = (-\sqrt{3}, 1)$ is on the terminal side of angle $\theta$, sketch the reference triangle, give the values of all 6 trig functions and determine the value of the reference angle and $\theta$.

Example 2:

Quadrantal Angles

Angles along the $x$ or $y$ axis

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$\frac{\pi}{2}$</th>
<th>$\pi$</th>
<th>$\frac{3\pi}{2}$</th>
<th>$2\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sin(\theta)$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>$\cos(\theta)$</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>$\tan(\theta)$</td>
<td>undefined</td>
<td>0</td>
<td>undefined</td>
<td>0</td>
</tr>
</tbody>
</table>

Example 3:

Circular Functions

For a unit circle, if $\theta$ is equal to the arc length $s$

Since $r = \sqrt{x^2 + y^2} = 1$

$\sin(\theta) = \sin(s) = y$
$\cos(\theta) = \cos(s) = x$
$\tan(\theta) = \tan(s) = \frac{y}{x}$

Thus trig functions are defined for lengths, quantities, or any real number $s$; trig functions are now functions of a real variable (and not just angles).

Trigonometric Functions of Real Numbers

Let $x$ be any real number and let $P(u, v)$ be the point corresponding to $x$ when the number line wrapped around the unit circle $u^2 + v^2 = 1$ then

$\sin(x) = v$
$\cos(x) = u$
$\tan(x) = \frac{v}{u}$
$csc(x) = \frac{1}{v}$
$sec(x) = \frac{1}{u}$
$\cot(x) = \frac{u}{v}$

A function $y = f(x)$ is periodic if there is a positive number $c$ such that $f(x + c) = f(x)$ for all $x$ in the domain. The smallest number $c$ is called the period.
Revisiting/Revising Two Famous Triangles

isosceles
45-45-90

\[
\begin{align*}
\triangle & \quad \text{with sides} \quad 1, 1, \sqrt{2} \\
& \quad \text{angles} \quad 45^\circ, 45^\circ, 90^\circ \\
& \quad \text{side ratios} \quad 1 : 1 : \sqrt{2}
\end{align*}
\]

equilateral
30-60-90

\[
\begin{align*}
\triangle & \quad \text{with sides} \quad 1, \sqrt{3}, 2 \\
& \quad \text{angles} \quad 30^\circ, 60^\circ, 90^\circ \\
& \quad \text{side ratios} \quad 1 : \sqrt{3} : 2
\end{align*}
\]

The 16-Point Unit Circle

Graphing sine(x)

Using the 16 point unit circle sketch the graph of sine(x)

Graphing cosine(x)

Using the 16 point unit circle sketch the graph of cosine(x).

Written Homework #29 – Due W 11/20/2013

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