Math 120: Elementary Functions
Today's Overview: The Other Trig Functions

y = tan(x)
- graph and its properties

y = secant(x)
- reciprocal of cos(x)
- graph and its properties

y = cot(x) and y = csc(x)
- One-to-one and inverse functions

Inverse Trig Functions

The 16-Point Unit Circle

Properties of y = tan(x)
- Sketching the graph
- Domain and range:
- Periodicity
- Symmetry
- Relative maximums and minimums
- Intervals of increase and decrease

The Other Four
- Secant – reciprocal of cosine: domain & range, period, x-intercepts, symmetry, asymptotes
- Cotangent – reciprocal of tangent
- Cosecant – reciprocal of sine

Evaluate the following
- a. \( \sec x = \frac{2}{\sqrt{3}} \)
- b. \( \cot x = \frac{\sqrt{3}}{3} \)
- c. \( \csc x = 2 \)
Review: 1:1 & Inverse Functions

A function \( f: A \rightarrow B \) is one-to-one if and only for every \( y \) in the range of \( f \), there is a unique \( x \) in the domain \( A \) such that \( f(x) = y \). That is if \( f(x_1) = y = f(x_2) \) then \( x_1 = x_2 \).

If \( f: A \rightarrow B \) is a one-to-one function then there is an inverse function \( f^{-1}: \text{Range}(f) \rightarrow A \) defined as follows: \( f^{-1}(b) = a \) if and only if \( f(a) = b \) for all \( b \) in Range(\( f \)).

Recall given a 1:1 function (e.g. \( f: A \rightarrow B \)) how to determine the inverse function.

\[
\begin{align*}
\text{If } & y = \frac{2x - 1}{x + 3} \\
\text{then } & y = \frac{1}{2}.
\end{align*}
\]

Swap \( x \) & \( y \) and solve for \( y \).

\( y = \sin^{-1}(x) = \arcsin(x) \)

\( \sin(x) \) restricted to the interval \([-\pi/2, \pi/2]\) is one-to-one (check out the graph). Therefore

Defn: \( y = \sin^{-1}(x) \) if and only if \( x = \sin(y) \) where \(-\pi/2 \leq y \leq \pi/2\) and \(-1 \leq x \leq 1\)

Example: Evaluate \( \sin^{-1}(1/2) \) using the definition.

\( y = \sin^{-1}(1/2) \) iff \( 1/2 = \sin(y) \) where \(-\pi/2 \leq y \leq \pi/2\)

So \( y = \pi/6 \) since \( \sin(\pi/6) = 1/2 \)

Example: Evaluate \( \sin^{-1}(-\sqrt{3}/2) \).

\( y = \cos^{-1}(x) = \arccos(x) \)

\( \cos(x) \) restricted to the interval \([0, \pi]\) is one-to-one (check out the graph). Therefore

Defn: \( y = \cos^{-1}(x) \) if and only if \( x = \cos(y) \) where \( 0 \leq y \leq \pi \) and \(-1 \leq x \leq 1\)

Example: Evaluate \( \cos^{-1}(1/2) \) using the definition.

\( y = \cos^{-1}(1/2) \) iff \( \cos(y) = 1/2 \) for \( 0 \leq y \leq \pi \)

So \( y = \pi/3 \) since \( \cos(\pi/3) = 1/2 \)

More Examples

Carefully evaluate the following

\[
\begin{align*}
\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) & & \sin^{-1}\left(\frac{5\pi}{6}\right) & & \sin^{-1}\left(\frac{\pi}{9}\right)
\end{align*}
\]

In general: \( \sin^{-1}(\sin(y)) = y \) if and only if \( -\pi/2 \leq y \leq \pi/2 \)
otherwise find the corresponding reference angle.

Important! \( \sin^{-1}(x) = \frac{1}{\sin(x)} = \csc(x) \)

Try \( \sin^{-1}\left(\frac{7\pi}{9}\right) \)

Do not confuse the inverse function with the reciprocal function.

Use \( \sin(x)^{-1} = \left(\sin(x)^{-1}\right)^{-1} \) for the reciprocal.

Written Homework #32 – Due M 12/03/2013

From page 365
29, 31, 33 (use reference triangles)
45, 46

From page 385
1, 9,
25, 29,
41, 43

Note: Because of periodicity, none of the trig functions are one-to-one. However if we restrict their domains, they can be one-to-one on the restricted domain.

Inverse sine function
\( y = \sin^{-1}(x) \) iff \( x = \sin(y) \) where \(-\pi/2 \leq y \leq \pi/2\) and \(-1 \leq x \leq 1\)

Inverse cosine function
\( y = \cos^{-1}(x) \) iff \( x = \cos(y) \) where \( 0 \leq y \leq \pi \) and \(-1 \leq x \leq 1\)

Inverse tangent function
\( y = \tan^{-1}(x) \) iff \( x = \tan(y) \) where \(-\pi/2 < y < \pi/2 \) and \(-\infty \leq x \leq \infty\)