1. **Limits (4 pts):** Complete the following definition of limit: A function \( f(x) \) has a limit \( L \) as \( x \) approaches \( c \), written \( \lim_{x \to c} f(x) = L \) if and only if …

2. **Limits (18 pts):** Find the indicated limit if it exists; other use DNE for Does Not Exist. Show any algebraic manipulations used to obtain the limit. Use your grapher only to verify! Repeat \( \lim_{x \to c} \) until the point where you actually take the limit at the end.

   a. \( \lim_{x \to 3} \frac{x - 3}{x^2 - x - 6} = \)

   b. \( \lim_{x \to 2} \frac{2x + 3}{x - 2} = \)

   c. \( \lim_{x \to 0} \frac{5x^2 + 3x + 4}{2x + 1} = \)

   d. \( \lim_{x \to 1} \frac{x - 1}{x} = \)

   e. \( \lim_{x \to \infty} \frac{3x^2 + 5x}{2x^2 + x + 1} = \)

   f. Evaluate the LHL \( \lim_{x \to 2^-} \frac{x + 2}{x - 2} = \)
3  Continuity (6 pts): Informally we say a function is continuous if its graph has no holes or breaks. Give the (3 point) formal definition of continuity of a function $f(x)$ at a point $c$.

b. Using the above definition determine if $f(x) = \begin{cases} x+1 & \text{if } x < 1 \\ -x^2 + 4x - 1 & \text{if } x \geq 1 \end{cases}$ is continuous at $x = 1$. Justify your answer to receive credit.

4  (13 pts) Complete the following definition of derivative: The derivative of a function $f(x)$ with respect to $x$ is the function $f'(x)$ given by …

b. Using the above definition calculate the derivative of the function $f(x) = 2x^2 - 5x + 1$. Be sure to carry through the “lim” until you actually evaluate the limit. Show all work

c. Using the above find the equation of the line tangent to the curve $f(x)$ above at $x = 3$. Express in slope intercept form.
5. **(30 pts)** Find the derivatives \( \frac{dy}{dx} \) for the following functions. Show all work.

a. \[ y = 5x^4 + 3x^2 - 7x + 6 + 2\sqrt{x} \]

b. \[ y = (3x + 2)(x^2 - 5x + 1) \] Reduce your answer

c. \[ y = \frac{1}{x} + \frac{3}{x^2} + \frac{4}{x^4} \]

d. \[ y = \frac{x^2 + 1}{x^2 + 2} \] Reduce your answer.

e. Find \( \frac{dy}{dx} \) for the implicit function \[ x^2 - y^2 = xy + 2 \]

f. \[ y = (4x - x^3)^2 \]
6. **(6 pts)** Find the coordinates (x and y) where the line tangent to the curve \( f(x) = \frac{4x}{x^2 + 1} \) is horizontal. Show all work. You may not use a calculator to solve this.

7. **(6 pts)** Chain Rule: If \( y = 3u^2 - u \) and \( u = x^2 - x - 1 \) find the value of \( \frac{dy}{dx} \) when \( x = 2 \). Show all work.

8. **(6 pts)** Find the equation of the line tangent to the curve \( x^2 - y^2 = 2x - 4y \) at the point \((2, 4)\). Express in slope-intercept form.

9. **(6 pts)** If the total cost \( C \) for producing \( x \) units of an item is given by the equation \( C(x) = 0.1x^3 + 5x + 120 \) find the marginal cost for producing the 11th unit and find the actual cost for producing the 11th unit.
10. **(6 pts) Using Newton’s Iteration Method** \( x_i = x_0 - \frac{f(x_0)}{f'(x_0)} \) find the zero for the function \( f(x) = x^3 - 4x - 5 \) using the initial value \( x_0 = 3 \). Write down the equation for Newton’s Method for the given function then using your calculator and give the first three values obtained from the iteration formula. Show all digits.

Write down Newton’s Formula for this function:

\[
\begin{align*}
x_1 &= \quad \text{__________________________} \\
x_2 &= \quad \text{__________________________} \\
x_3 &= \quad \text{__________________________}
\end{align*}
\]

**Extra Credit**: 10 pts - Evaluate the following limits. Show all work to receive credit.

a. Complete the statement of the Intermediate Value Theorem: If \( f(x) \) is *continuous* on the closed interval \([a, b]\) and if \( L \) is a value between \( f(a) \) and \( f(b) \) then there is a point \( c \) between \( a \) and \( b \) such that …

b. Evaluate \( \lim_{h \to 0} \frac{\sqrt{x + h} - \sqrt{x}}{h} = \)

c. Find \( \frac{dy}{dx} \) for \( y = \left( \frac{x+1}{x-2} \right)^3 \)