Math 131: Essentials of Calculus  
Spring 2010 - Review III

**FOIL – First Outside Inside Last**  
Appendix A.2 page 670

Polynomial Multiplication (see below)

**Binomial Expressions/Expansions: \((a+b)^n\) for positive integer n**

1. Trivially (using FOIL) \((a+b)^2 = a^2 + 2ab + b^2\)

2. \((a+b)^3 = (a+b)^2(a+b) = a^2 + 2ab + b^2 \times a + b\)

\[
\begin{align*}
a^3 + 2a^2b + ab^2 \\
a^3 + 3a^2b + 3ab^2 + b^3
\end{align*}
\]

3. \((a+b)^4 = (a+b)^3(a+b) = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4\) obtainable by multiplying \((a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3\) by \(a+b\)

4. There are two patterns here:

   1. When expanding \((a+b)^n\) the exponents for \(a\) start at \(n\) and decrease to 0 while the exponents for \(b\) start at 0 and increase to \(n\). For all terms the sum of the exponents is always \(n\).

   2. The coefficient for each term can be obtained from Pascal’s triangle

\[
\begin{array}{cccccc}
1 & & & & & \\
1 & 1 & & & & \\
1 & 2 & 1 & & & \\
1 & 3 & 3 & 1 & & \\
1 & 4 & 6 & 4 & 1 & \\
1 & 5 & 10 & 10 & 5 & 1 \\
1 & 6 & 15 & 20 & 15 & 6 & 1 \\
\end{array}
\]

where each entry is obtained by summing the two entries above it.

**Factoring:**  
Appendix A: pages 671 - 673

1. Trial and Error/Guess and Check

To factor \(ax^2 + bx + c\) start with the factors of \(ax^2\) and the factors of \(c\), guess a solution, the check by FOIL-ing.
Example: Factor $6x^2 - 4x - 2$.

The factors of $6x^2$ are $6x$, $x$ and $3x$, $2$ (also $-6x$, $-x$ and $-3x$, $-2$). The factors of $-2$ are $-2$, $1$ and $2$, $-1$. Guess $(3x-1) \cdot (2x+2) = 6x^2 + 6x - 2x - 2 = 6x^2 + 4x - 2$. Not correct by close so try $(3x+1) \cdot (2x-2) = 6x^2 - 6x + 2x - 2 = 6x^2 - 4x - 2$.

2. Beware the following CMS: $\sqrt{a^2 + b^2} = a + b$ Why?

**Solving (Quadratic) Equations by Factoring:** Appendix A: page 675

**Completing the Square:** Appendix A: page 676

1. For reasons to be given later, it is useful to re-write the quadratic equation $y = ax^2 + bx + c$ into the form $y = a(x-h)^2 + k$. Do the following.

   A. Using FOIL expand $a(x-h)^2 + k = ax^2 - 2ahx + (ah^2 + k)$

   B. Set $ax^2 + bx + c$ equal to $ax^2 - 2ahx + (ah^2 + k)$.

   C. Equating coefficients yields
      
      \[ a = a \]
      \[ b = -2ah \]
      \[ c = ah^2 + k \]

   D. Solve for $h = \frac{b}{-2a}$ and $k = c - \frac{b^2}{4a} = -\frac{b^2 - 4ac}{4a}$

**Example:**

$6x^2 - 4x - 2 = a(x-h)^2 + k$ 
Equating coefficients yields $-4 = -2ah$ Therefore $-2 = ah^2 + k$

$a = 6$ 

$h = \frac{1}{3}$ or $6x^2 - 4x - 2 = 6\left(x - \frac{1}{3}\right)^2 - \frac{8}{3}$. Check the answer by FOIL-ing $6\left(x - \frac{1}{3}\right)^2 - \frac{8}{3}$!
**Vertex Form of Quadratic:** \( f(x) = a(x-h)^2 + k \)

1. If \( a > 0 \) then the parabola opens up; if \( a < 0 \) then the parabola opens down

2. The vertex is the point \((h, k)\); this is the minimum (lowest) point if \( a > 0 \) (parabola opens up) or the maximum (highest) point if \( a < 0 \) (parabola opens down)

3. \( x = h \) is the axis of symmetry

4. If the parabola opens up \( (a > 0) \) and the vertex is below the x-axis \( (k < 0) \) then the roots/zeros of the parabola can be obtained by solving the vertex form \( a(x-h)^2 + k = 0 \) to obtain

\[
x = h \pm \sqrt{\frac{-k}{a}} = \frac{-b}{2a} \pm \sqrt{\frac{-4ac-b^2}{4a^2}} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}
\]

(note the quadratic formula). If the parabola opens down \( (a < 0) \) and the vertex is above the x-axis \( (k > 0) \) then the roots/zeros of the parabola can be obtain the same way.

**Graphing Parabolas** Chapter 1.2: page 20 - 21

**Quadratic Formula** Appendix A: page 677

Before you are allowed to graduate from Wittenberg you must correctly give the secret incantation. This is done at graduation. When you walk across the stage to receive your diploma, before you shake the president’s hand he will say “State the quadratic formula”. You must reply “ex equals minus be plus or minus the square root of be squared minus four ay cee over two ay”; otherwise you will not be allowed to graduate.

**Homework:** Page 680

**Problems**

<table>
<thead>
<tr>
<th>Problems</th>
<th>Instructions</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 – 10:</td>
<td>FOIL or polynomial multiplication</td>
</tr>
<tr>
<td>29 – 58</td>
<td>choose every 3rd or 4th problem</td>
</tr>
<tr>
<td>59 – 68</td>
<td>solve by factoring</td>
</tr>
<tr>
<td>75 – 82</td>
<td>complete the square to obtain the vertex form them solve for the zeros; using vertex, symmetry and zeroes (if they exist) sketch the graph</td>
</tr>
<tr>
<td>83 – 88</td>
<td>quadratic formula</td>
</tr>
</tbody>
</table>

**Plus …**

Using Pascal’s Triangle expand the following binomials

a. \((x + y)^6\)  
b. \((a + b)^7\)  
c. \((x+1)^5\)  
d. \((x+h)^5\)

**Review Quiz #3** Friday – All Questions taken from above