Math 131: Essentials of Calculus  
Spring 2010 - Review I

Intervals:  Appendix A: page 660

1. Be able to convert back and forth between inequalities (e.g. \( a < x < b \)), interval notation \((a, b)\) and line segments on a number line.

2. Convention: parenthesis \((\ )\) do not include the endpoints, square brackets \([\ ]\) do include the endpoints.

3. Infinity \(\infty\) is not a number and is not a point. Therefore the inequality \(a < x\) in interval notation is \((a, \infty)\) and \(a \leq x\) is \([a, \infty)\). Both \((a, \infty]\) and \([a, \infty]\) are WRONG and are examples of CMS’s (Common Stupid Mistakes!). By the way, \((\infty, a)\) and \((\infty, a]\) are also CSM’s.

Absolute Value:  Appendix A: page 661

1. The definition of absolute value \(|x|\) is an example of a piece-wise defined function.

Distance:  
\[ |x - a| < d \text{ if and only if } -d < x - a < d \]

1. \(|x - a| < d\) is the set of all points \(x\) within \(d\) units of point \(a\).

2. Visually think of this as an open interval centered at \(a\). You want all values \(x\) within distance \(d\) of \(a\) – so “move” \(d\) to the left (-\(d\)) and \(d\) to the right (+\(d\)) and this gives the interval \((a - d, a + d)\)

\[
\begin{array}{c}
-\text{d} \\
\hline
\text{a - d} & \text{a} & \text{a + d} \\
\hline
+\text{d}
\end{array}
\]

Therefore \(a - d < x < a + d\) or \(-d < x - a < d\)

3. \(|x - a| > d\) is the set of all point outside of \(d\) units from point \(a\).

4. Visually think of this as everything outside the closed interval (a close interval includes the endpoints) centered at \(a\). You want all values \(x\) greater than distance \(d\) from \(a\) so “move” more than \(d\) to the left (-\(d\)) and more than \(d\) to the right (+\(d\)) This gives two intervals \((-\infty, a - d)\) or \((a + d, \infty)\). It’s **or not and** because you’re in one or the other interval, not both.

\[
\begin{array}{c}
\text{a - d} \\
\hline
\text{a} & \text{a + d} \\
\hline
\end{array}
\]

Therefore \(x < a - d\) or \(a + d < x\) or \((-\infty, a - d)\) or \((a + d, \infty)\) \(\equiv (-\infty, a - d) \cup (a + d, \infty)\)

To say \(a + d < x < a - d\) is a CSM! Why?
Exponents and Roots  

1. Definition: For positive integer $n$, $a^n = a \cdot a \cdot a \cdots a$, i.e. $a$ multiplied by itself $n$ times.  

Therefore it follows that $\frac{a^n}{a^m} = \frac{a \cdot a \cdot a \cdots a}{a \cdot a \cdots a} = a^{n-m}$ by cancellation. 

2. If $m > n$ then a negative exponent means the $a$'s appear in the denominator; hence $a^{-n} = \frac{1}{a^n}$. 

3. If $n = m$ all the $a$'s cancel leaving the value 1; hence $a^n = 1$. 

4. Since $(\sqrt[n]{a})^2 = (\sqrt[n]{a}) \cdot (\sqrt[n]{a}) = a^1$ we can extend the pattern to include fractional exponents; hence $a^{\frac{1}{n}} = \sqrt[n]{a}$ and in general $a^{\frac{1}{n}} = \sqrt[n]{a}$ for any positive integer $n$. 

5. With negative values the pattern dictates that $a^{-\frac{1}{n}} = \frac{1}{\sqrt[n]{a}}$. 

6. For non-integral fractions like $\frac{n}{m}$ then $a^{\frac{n}{m}} = (\sqrt[n]{a})^n = \sqrt[n^m]{a^n}$ (provided you stay away from negative values of $a$). 

Rationalizing  

1. Recall $(a+b) \cdot (a-b) = a^2 - b^2$. Hence $(\sqrt{x} + \sqrt{y}) \cdot (\sqrt{x} - \sqrt{y}) = x - y$ 

Note that $(\sqrt{x} + \sqrt{y})$ and $(\sqrt{x} - \sqrt{y})$ are the conjugates of each other. 

Homework: page 667 

Problems 

Instructions: Do as many from each set until mastery is achieved 

1 – 4: line segments to inequalities and interval notation 
5 – 8: inequalities to line segments and interval notation 
13 – 18: express in interval notation 
19 – 26: convert fractional exponents to radicals then solve 
27- 34: reduce then evaluate 
35 – 42: use the obvious fact that if $a^n = a^m$ then $n = m$ 
43 – 76 do every 3rd or 4th 
79, 80, 83 - 86 rationalizing denominators 

Review Quiz #1 Friday – All questions taken from above