Math 131: Essentials of Calculus
Review Sheet – Test #3

Ch 5.1 – Anti-differentiation

Fundamental Property of Anti-derivatives (page 373); Four Functional Rules for Integration (page 375); Algebraic Rules (page 376); Initial Value Problems (page 378)

\[ a) \int \left(2x^5 + 8x^3 - 3x^2 + 5\right) dx \]

Find the following integrals (Ex 5.1.3)

\[ b) \int \left(\frac{x^3 + 2x + 7}{x}\right) dx \]

\[ c) \int 3e^{-5t} + \sqrt{t} dt \]

Find the function \( f(x) \) whose tangent has slope \( 3x^2 + 1 \) for each value of \( x \) and whose graph passes through the point \((2, 6)\) (Ex 5.1.4)

Ch 5.2 – u-substitution

Find

\[ \int \sqrt{2x+7} \, dx \text{ (Ex. 5.2.2)}, \quad 8x \left(4x^3 - 3\right)^5 \, dx \text{ (Ex. 5.2.2)}, \quad x^3 e^{x^2+2} \, dx \text{ (Ex. 5.2.3)}, \]

\[ \int \frac{x}{x+1} \, dx \text{ (Ex. 5.2.4)}, \quad \text{and} \int \frac{x^3+3x+5}{x+1} \, dx \text{ (Ex. 5.2.8)} \]

Ch 5.3 – The Definite Integral and the Fundamental Theorem of Calculus

Know the definition of the Riemann Sum (page 401); Know the definition of the Definite Integral (page 401); Know and be able to state Fundamental Theorem of Calculus (page 402); Rules of Definite Integrals (page 404)

Exercise: Make a sketch of the function \( f(x) = \frac{1}{x} + x^2 \) on the interval \([1,2]\). Write out and compute the value of the Riemann sum using 4 sub-intervals with inscribed rectangles, and using 8 subintervals with circumscribed rectangles. Show all work.

Evaluate \[ \int_{0}^{1} \left(e^{-x} + \sqrt{x}\right) dx \text{ (Ex. 5.3.4)}; \quad \text{Evaluate} \int_{0}^{1} 8x \left(x^2 + 1\right)^3 \, dx \text{ (Ex. 5.3.7)} \]

Also check out Example 5.3.6

Ch 5.4 – Applying Definite Integration

A Procedure for Using Definite Integration in Applications (page 414)
Standard Applications of Definite Integrals – from Chapters 5.3, 5.4 and 5.6

A. Area under a Curve – See Example 5.3.3.
B. Net Change (page 408) – See Example 5.3.9
C. Area between Two Curves (page 417) – See Example 5.4.2
D. Average Value of a Function (page 424) – See Example 5.4.6

Note the Rate Interpretation of Average Value (page 426) and the Geometric Interpretation of Average Value (page 427)

E. Volume of Revolution (page 454) – See Example 5.6.6

Ch 6.1 – Integration by Parts

Evaluate \( \int xe^{2x} \, dx \) (Ex. 6.1.2); \( \int x\sqrt{x+5} \, dx \) (Ex. 6.1.3) \( \int \ln(x) \, dx \) (Ex. 6.1.4) \( \int x^2 e^{-2x} \, dx \) (Ex. 6.1.6)