Math 171 – Discrete Mathematical Structures

Today’s Overview

1. Logical Forms and Logical Equivalence
2. Statements
   compound statements, and: \( \land \), or: \( \lor \), truth values \( T, F \)
3. Truth Tables
4. Logical Equivalence
   De Morgan’s Laws, tautologies, contradictions, Theorem 2.1.1 on Logical Equivalences

Logical Forms and Logical Equivalence

Argument form: sequence of statements aimed at demonstrating the truth of an assertion (conclusion) from preceding statements (premises) which are assumed true (hypotheses) or logically derived from early premises.

Distinguish form from content.

Validity of argument follows from form – the truth of the conclusion necessarily follows from truth of premises.

Example

If the program syntax is faulty or the program execution results in division by zero, the program will not execute correctly.
The program executed correctly.
Therefore the program syntax was not faulty and the program execution did not divide by zero.

if \( p \) or \( q \) then \( r \)
not \( r \)
not \( p \) and not \( q \)

Statements

A statement (proposition) is a sentence which is true or false.

Compound Statements (not, and, or)

Not \( p \), symbolically \( \neg p \)
or \( \sim p \)
the negation of \( p \) flips the truth value of \( p \)

\( p \) and \( p \), symbolically \( p \land q \)
the conjunction of \( p \) and \( q \) is true iff both \( p \) and \( q \) are true

\( p \) or \( q \), symbolically \( p \lor q \)
the disjunction of \( p \) and \( q \) is true iff either \( p \) is true or \( q \) is true or both (inclusive or and not the exclusive or)

Aside: exclusive or is \( (p \lor q) \land \neg(p \land q) \) OR \( (p \land \neg q) \lor (\neg p \land q) \)

Order of operations: 1: parentheses; 2: \( \neg \); 3: \( \land \); 4: \( \lor \)

Translating English to Symbols

p but \( q \) is equivalent to \( p \) and \( q \); that is \( p \land q \)
neither \( p \) nor \( q \) is equivalent to not \( p \) and not \( q \); that is \( \neg p \land \neg q \)

Truth Values and Truth Tables

\[
\begin{array}{c|c|c|c|c|c|c|c}
| p | \sim p | & q | p \land q | & p | q | p \lor q |
|---|---|---|---|---|---|
| T | F | T | T | T | T |
| T | T | F | T | T | T |
| F | T | T | F | F | T |
| F | T | F | F | F | F |
\end{array}
\]

Note: arrangement of T’s and F’s in columns
Tautologies & Contradictions

A tautology is a statement form that is always true.
Example: \( p \lor \sim p \)

A contradiction is a statement form that is always false.
Example: \( p \land \sim p \)

Theorem 2.1.1: Logical Equivalences

Commutative
\[ p \land q \equiv q \land p \]
\[ p \lor q \equiv q \lor p \]

Associative
\[ (p \land q) \land r \equiv p \land (q \land r) \]
\[ (p \lor q) \lor r \equiv p \lor (q \lor r) \]

Distributive
\[ p \land (q \lor r) \equiv (p \land q) \lor (p \land r) \]
\[ p \lor (q \land r) \equiv (p \lor q) \land (p \lor r) \]

Identity
\[ p \land T \equiv p \]
\[ p \lor F \equiv p \]

Negation
\[ p \land \sim p \equiv \bot \]
\[ p \lor \sim p \equiv \top \]

Double Negative
\[ \sim \sim p \equiv p \]

“duals”: swap \( \land \) & \( \lor \), \( T \) and \( F \)

Theorem 2.1.1: Logical Equivalences (cont.)

Idempotent
\[ p \land p \equiv p \]
\[ p \lor p \equiv p \]

Universal bound
\[ p \land F \equiv F \]
\[ p \lor T \equiv T \]

De Morgan
\[ \sim (p \land q) \equiv \sim p \lor \sim q \]
\[ \sim (p \lor q) \equiv \sim p \land \sim q \]

Absorption
\[ p \land (p \lor q) \equiv p \]
\[ p \lor (p \land q) \equiv p \]

Using Logical Equivalence to Simplify Statement Forms

Prove: \( (p \lor q) \land (p \lor q) \equiv p \)

\[ (p \lor q) \land (p \lor q) \equiv p \lor (\sim q \land q) \]
\[ \equiv p \lor (q \land \sim q) \]
\[ \equiv p \lor F \]
\[ \equiv p \]

Written Homework #3 – Due W 9/4/13

Exercises Set 2.1 (p. 37) #22, #37, #41, #49, #54

Using a truth table prove the other De Morgan Law
\[ \sim (p \lor q) \equiv \sim p \land \sim q \]