Math 171 – Discrete Mathematical Structures

Today’s Overview

Negatives of Quantified Expressions

Negations of Universal and Existential Conditional Expressions

Negation of Quantified Expressions:

- \((\forall x \in D, Q(x)) \equiv \exists x \in D, \lnot Q(x)\)

If it is not the case that for all \(x \in D\), that \(Q(x)\) is true then there must be some \(a \in D\) such that \(Q(a)\) is false. Thus \(\exists x \in D, \lnot Q(x)\)

Negations of quantified conditional statements

Not all cars are red \(\equiv\) There is a car which is not red

It is not that case that for all integers, if \(x\) is prime then \(x\) is odd \(\equiv\) there is an integer that is prime and not odd.

DeMorgan’s Law

Quantified Statements on Finite Domains – DeMorgan’s Law

If \(D = \{a_1, a_2, \ldots, a_n\}\) then \(\forall x \in D, P(x)\) is equivalent to

\[ P(a_1) \land P(a_2) \land \cdots \land P(a_n) \]

and \(\exists x \in D, P(x)\) is equivalent to

\[ P(a_1) \lor P(a_2) \lor \cdots \lor P(a_n) \]

Therefore

\[ - (\forall x \in D, P(x)) \equiv - (P(a_1) \land P(a_2) \land \cdots \land P(a_n)) \equiv \exists x \in D, \lnot P(x) \]

\[ - (\exists x \in D, P(x)) \equiv - (P(a_1) \lor P(a_2) \lor \cdots \lor P(a_n)) \equiv \forall x \in D, \lnot P(x) \]

Note:

- \((\exists x, P(x) \rightarrow Q(x)) \neq \forall x, P(x) \rightarrow Q(x)\)

Vacuous Truth of Universal Statements: \(\forall x \in D, P(x) \rightarrow Q(x)\) is vacuously (or trivially) true if \(P(x)\) is false for all \(x \in D\).

Note the negation: \(\exists x \in D, P(x) \land \lnot Q(x)\) is false (why?)

Contrapositive, Converse, and Inverse forms

\[ \forall x, Q(x) \rightarrow P(x) \iff \forall x, P(x) \rightarrow \lnot Q(x) \]

Necessary \((r(x))\) is necessary for \(s(x)\) or \(\forall x, r(x) \rightarrow \lnot s(x)\) and sufficient \((r(x))\) is sufficient for \(s(x)\) or \(\forall x, r(x) \rightarrow s(x)\) conditions

Written Homework #8 – Due M 9/16/13

Exercise Set 3.2 (p. 115) #15, #16 - #19, #26 - #29