Math 171 – Discrete Mathematical Structures
Today’s Overview
Assumptions & Number Theory Definitions
Proving Existential Statements
Disproving Universal Statements by Counterexample
Proving Universal Statements

Assumptions
Laws of Basic Algebra (see Appendix A)
Three Properties of Equality
1) A = A
2) If A = B then B = A
3) If A = B and B = C then A = C
There are no integers between 0 and 1
The set of integers is closed under addition, subtraction and multiplication

Number Theory Definitions
An integer \( n \) is even iff \( \exists k \in \mathbb{Z} \) such that \( n = 2k \)
An integer \( n \) is odd iff \( \exists k \in \mathbb{Z} \) such that \( n = 2k + 1 \)
An integer \( n \) is prime iff \( \forall r, s \in \mathbb{Z} \), if \( n = rs \) then \( r = 1 \) or \( s = 1 \)
An integer \( n \) is composite iff \( \exists r, s \in \mathbb{Z} \) such that \( n = rs \) and \( 1 < r < n \) and \( 1 < s < n \)

Proving Existential Statements:
Find an example (constructive proof of existence)
Example: There exists a prime \( p \) such that \( 2^p - 1 \) is prime

Disproving Universal Statements:
Find a counter-example (proof by counter example)
Example: For all primes \( p \), \( 2^p - 1 \) is prime

Existential Instantiation: If the existence of a certain kind of object is assumed or has been deduced then it can be given a name as long as that name is not currently being used to denote something else.

Showing an existential statement \( \exists x \in D, Q(x) \) is false
\[- \exists x \in D, Q(x) \iff \forall x \in D, \neg Q(x) \]
That is, show \( Q(x) \) is false for all \( x \) in the domain.

Example: Show the following is false: \( \exists n \in \mathbb{Z}^+, n^2 + 3n + 2 \) is prime

What if \( n = 0 \)?
Written Homework #11 – Due M 9/23/13

Exercise Set 4.1 (page 161) #10, #18, #25, #28, #29, #31,