Math 171 – Discrete Mathematical Structures

Today’s Overview

Quotient – Remainder Theorem
div and mod
Representing Integers
Absolute Value and Triangle Inequality

Quotient Remainder Theorem

Given any integer \( n \) and positive integer \( d \), there exist unique integers \( q \) and \( r \) such that

\[
n = q d + r \quad \text{where} \quad 0 \leq r < d
\]

Example:

If \( n = 17 \) and \( d = 5 \) then \( 17 = 3 \cdot 5 + 2 \)
If \( n = -17 \) and \( d = 5 \) then \( -17 = -4 \cdot 5 + 3 \)

div (\(/\)) and mod (%)

Python: remainder is same sign as divisor
Java, C, C++: remainder is same sign as dividend (rounded towards zero)

\[
n \mod d = n - d \cdot (n \div d)
\]

mod notation: e.g. \( 16 \mod 7 = 2 \iff 16 = 7 \cdot k + 2 \) for some integer \( k \)

Examples

9/25/13 is a Wednesday. What day of the week will 9/25/14 be?

Prove: If \( n \mod 7 = 2 \) and \( m \mod 7 = 3 \) then \( n \cdot m \mod 7 = 6 \)

Prove: If \( n \mod 7 = 2 \) then there is no integer \( k \) such that \( k \cdot n \mod 7 = 0 \) unless \( k \) is a multiple of 7. (Hint: use method of exhaustion)

Question. Is the same true for \( n \mod 6 = 2 \)? Is there an integer \( m \) which is not a multiple of 6 such that \( n \cdot m \mod 6 = 0 \)?

Prove: The product of two consecutive integers is even

Representing Integers

Odd vs. Even or \( 2k + 1 \) vs. \( 2k \) integers

Since every odd prime \( p \) is in the form \( 4k + 1 \) or \( 4k + 3 \) for some integer \( k \), prove \( p^2 = 4k^2 + 1 \) for some integer \( k \)

Prove (see textbook): if \( n \) is any odd integer then \( n^2 = 8k + 1 \) for some integer \( k \)

Absolute Value and the Triangle Inequality

Absolute Value:

\[
| x | = \begin{cases} 
 x & \text{if} \quad x \geq 0 \\
-x & \text{if} \quad x < 0 
\end{cases}
\]

Triangle Inequality: For all real numbers \( x \) and \( y \), \( |x + y| \leq |x| + |y| \)
Written Homework #13 – Due M 9/30/13

Exercise Set 4.3 (page 179) #44, #45, #46

Exercise Set 4.4 (page 189) #19, #20, #23, #25, #29, #43