Math 171 – Discrete Mathematical Structures

Today’s Overview

Induction vs. Deduction

More Mathematical Induction

Induction vs. Deduction

Induction – obtaining a general principle from specific instances; i.e. arguing from specific cases to the general case

Deduction: inferring a conclusion from general principles.

Example: Observe

| n | 1 | 1 | 1 | 1 | 2 | 3 | 3
|---|---|---|---|---|---|---|---
| 2n+1 | 2 | 1 | 2 | 1 | 2 | 1 | 2

Infer a general rule (induction) and prove it using mathematical induction (deduction).

Proving Inequalities using Mathematical Induction

Prove \(2n+1 < 2^n\) for \(n \geq 3\)

Base Case: \(n = 3\) show \(2(3) + 1 < 2^3\)

Induction Hypothesis: Assume for \(n > 3\), \(2n+1 < 2^n\)

Induction Step: Prove \(2(n+1) + 1 < 2^{n+1}\)

Proof: \(2(n+1) + 1 = 2n + 2 + 1 = (2n + 1) + 2 < 2^n + 2\)

But for \(n > 3\), \(2 < 2^n\) so \(2^n + 2 < 2^n + 2^n = 2^{n+1}\)

Therefore \(2(n+1) + 1 < 2^n + 2 < 2^{n+1}\). QED

Using Mathematical Induction to obtain closed formulas from recursive definitions

Given the recursive definition \(a_n = \begin{cases} 5 & \text{if } n = 0 \\ 2a_{n-1} & \text{if } n > 0 \end{cases}\) find and prove a closed form for \(a_n\).

Conjecture:

Base Case:

Induction Hypothesis:

Induction Step:

Proof:

Written Homework #17 – Due F 10/11/13

From Exercise Set 5.3 (p. 266): Indicate/Identify/Label the base case, the induction hypothesis, the induction step (i.e. what you’re trying to prove) and the proof itself for each exercise below:

#3, #8 (see #6), #16 (see #7), #19, #23

(note – problems in parentheses may help)