Math 171 – Discrete Mathematical Structures

Today’s Overview

Solving Recurrence Relations by Iteration
Arithmetic sequences
Geometric sequences
Using Mathematical Induction to Prove Correctness
Two useful summation formulas you should know (and can use)

The Method of Iteration

Problem: Given a recurrence relation – find a closed formula for the \( n \)th term – method of iteration

Example \( a_n = \begin{cases} 
2 & \text{if } n = 0 \\
 a_{n-1} + 3 & \text{if } n > 0
\end{cases} \)

\( a_0 = 2 \)
\( a_1 = a_0 + 3 = 2 + 3 \)
\( a_2 = a_1 + 3 = (2 + 3) + 3 \)
\( = 2 + (3 + 3) \)
\( a_3 = a_2 + 3 = (2 + (3 + 3)) + 3 \)
\( = 2 + (3 + 3 + 3) \)
\( a_4 = a_3 + 3 = (2 + (3 + 3 + 3)) + 3 \)
\( = 2 + (4 \\cdot 3) \)
\( a_5 = a_4 + 2 = (3 + 4 \\cdot 3) + 3 \)
\( = 2 + 5 \\cdot 3 \)

Guess? \( a_n = 2 + n \\cdot 3 \)

Arithmetic Sequence: a sequence of the form \( a_0, a_1, a_2, \ldots \) is called an arithmetic sequence if and only if there is a constant \( d \) such that \( a_n = a_{n-1} + d \) for all \( n \geq 1 \)

It follows that \( a_n = a_0 + n \cdot d \)

Example \( a_n = \begin{cases} 
1 & \text{if } n = 0 \\
 \frac{9}{10} a_{n-1} & \text{if } n > 0
\end{cases} \)

\( a_0 = 2 \)
\( a_1 = (9/10)a_0 = (9/10) \cdot 2 \)
\( = 2 \cdot (9/10) \)
\( a_2 = (9/10)a_1 = (9/10) \cdot (2 \cdot (9/10)) \)
\( = 2 \cdot (9/10)^2 \)
\( a_3 = (9/10)a_2 = (9/10) \cdot (2 \\cdot (9/10)^2) \)
\( = 2 \cdot (9/10)^3 \)

Guess: \( a_n = 2 \cdot (9/10)^n \)

Geometric Sequence: a sequence of the form \( a_0, a_1, a_2, \ldots \) is called a geometric sequence if and only if there is a constant \( r \) such that \( a_n = r \cdot a_{n-1} \) for all \( n \geq 1 \)

It follows that \( a_n = a_0 \cdot r^n \)

“And now for something completely different”

Try \( b_n = \begin{cases} 
1 & \text{if } n = 0 \\
 b_{n-1} + a_{n-1} & \text{if } n > 0
\end{cases} \)

Using Mathematical Induction to Prove correctness

Prove if \( a_n = \begin{cases} 
2 & \text{if } n = 0 \\
 a_{n-1} + 3 & \text{if } n > 0
\end{cases} \) then \( a_n = 2 + n \cdot 3 \)

Base case: \( n = 0 \)
\( 2 = a_0 = 2 + 0 \cdot 3 \)

Induction Hypothesis: for some \( n \geq 0 \) \( a_n = 2 + n \cdot 3 \)

Induction Step: Prove \( a_{n+1} = 2 + (n+1) \cdot 3 \)

Proof: \( a_{n+1} = a_n + 3 = (2 + n \cdot 3) + 3 = 2 + (n+1) \cdot 3 \)

Two Useful Formulas

Geometric Series: \( \sum_{k=1}^{n} r^k = \frac{r^{n+1} - 1}{r - 1} \)

\( \sum_{k=1}^{n} k = \frac{n(n+1)}{2} \)

Example: Find an explicit formula for TOH sequence

\( m_n = \begin{cases} 
1 & \text{if } n = 1 \\
 1 + 2m_{n-1} & \text{if } n > 1
\end{cases} \)
Example: A complete graph with \( n \) vertices, denoted \( K_n \), has an edge between each pair of vertices. Let \( e_n \) denote the number of edges in the complete graph \( K_n \).

1. Find a recurrence relation for \( e_n \).
2. Find an explicit closed formula for \( e_n \).

Written Homework – #20 Due W 10/23/13
Due F 10/25/13

From Exercise Set 5.7 (p. 314)
#1, #2, #3, & #28, #9 & #34, #26 a) – d) (Annuity)