Math 171 – Discrete Mathematical Structures
Today’s Overview

Set Theory: Set and “is element of”
Subsets
Set Equality
Set Operations: Union, Intersection, Difference, Complement
The Empty Set
Partitions of Sets
Power Sets
Cartesian Products

Sets, “is element of”, Subsets, Set Equality
set and element are undefined terms of set theory; a set is a “collection into a whole M of definite and separate objects of our intuition or thought. These objects are called elements of M.”

\[ a \in S \text{ means a is an element of } S; \quad a \notin S \text{ means a is not an element of } S \]

Subsets: \[ A \subseteq B \text{ if and only if } \forall x \in A \Rightarrow x \in B \]
Set Equality: \[ A = B \text{ iff } A \subseteq B \text{ and } B \subseteq A \]

Venn Diagrams: Sets A, B and C – Identify the 8 areas + contiguous combinations

Proving Set Containment
If \( A = \{ n \in Z \mid n = 4r + 1 \text{ for some } r \in Z \} \) and if \( B = \{ m \in Z \mid m = 2s + 1 \text{ for some } s \in Z \} \)
Show \( A \subseteq B \)
Let \( x \in A \)
\( \ldots \)
Then \( x \in B \). Therefore \( A \subseteq B \) QED!

Show \( B \subseteq A \) ([proof by counter-example])

Set Operations
Given sets A and B, subsets of a universal set U
Set Union \( A \cup B = \{ a \in U \mid a \in A \vee a \in B \} \)
Set Intersection \( A \cap B = \{ a \in U \mid a \in A \wedge a \in B \} \)
Set Difference \( A - B = \{ a \in U \mid a \in A \wedge a \notin B \} \)
Set Complement \( \overline{A} = \{ a \in U \mid a \notin A \} \)

Venn Diagram & Interval Examples
Indexed Union: \( \bigcup_{i=0}^{n} A_i = \{ a \in U \mid \exists k \leq n \wedge a \in A_k \} \)
Indexed Intersection: \( \bigcap_{i=0}^{n} A_i = \{ a \in U \mid \forall k \leq n \wedge a \in A_k \} \)

The Empty Set: \( \emptyset = \{ a \in U \mid a \notin U \} \); i.e. \( x \in \emptyset \) is false.
Note: For any set \( S \), \( \emptyset \subseteq S \) since \( \forall x (x \in \emptyset \Rightarrow x \in S) \)

Partitions of Sets
Two sets A and B are disjoint iff \( A \cap B = \emptyset \)
Sets \( A_1, A_2, \ldots, A_n \) are mutually (or pairwise) disjoint iff \( \forall i \neq j \Rightarrow A_i \cap A_j = \emptyset \)
A finite (or infinite) collection of non-empty sets \( (A_1, A_2, \ldots, A_n) \) is a partition of a set \( S \) iff

1. \( \bigcup_{i=1}^{n} A_i = S \)
2. the sets are mutually disjoint

Exercise: How many ways can you partition \( \{a, b, c\} \)?
Power Sets: Given a set $S$, the power set of $S$, $P(S)$ is the set of all subsets of $S$.

Note $\emptyset \in P(S)$

Cartesian Products: Given non-empty sets $A$ and $B$ define

$A \times B = \{(a, b) \mid a \in A \land b \in B\}$

Question: Given a finite non-empty set $A$, which is larger? $A \times A$ or $P(A)$

Written Homework – #21 Due F 10/25/13

From Exercise Set 6.1 (p. 349)

#3, #5, #9, #11, #15, #16, #25, #27, #35