Math 171 – Discrete Mathematical Structures

Today’s Overview

Subset "Relations"
Proofs for Sets
The Standard Set Identities
The Empty Set

Subset Relations (Theorem 6.2.1)

1. Inclusion of Intersection: For all sets A and B
   \[ A \cap B \subseteq A \] and \[ A \cap B \subseteq B \]

2. Inclusion in Union: For all sets A and B
   \[ A \subseteq A \cup B \] and \[ B \subseteq A \cup B \]

3. Transitive Property of \( \subseteq \): For all sets A, B and C
   If \( A \subseteq B \) and \( B \subseteq C \) then \( A \subseteq C \)

Procedure Versions of Set Definitions

Needed for proofs
\[
\begin{align*}
  x \in X \cup Y & \iff x \in X \lor x \in Y \\
  x \in X \cap Y & \iff x \in X \land x \in Y \\
  x \in X \setminus Y & \iff x \in X \land x \notin Y \\
  x \in X' (\text{or } x \notin X) & \iff x \notin X \\
  (x, y) \in X \times Y & \iff x \in X \land y \in Y
\end{align*}
\]

Example: Prove for all sets A and B
\[ A \cap B \subseteq A \]
Proof: Let \( x \in A \cap B \)
\[
\begin{align*}
  x & \in A \\
  x & \in B \\
  x & \in A \cap B \\
  \text{conjunctive simplification}
\end{align*}
\]

Therefore \( A \cap B \subseteq A \) definition of set containment

Set Identities

For all sets A, B and C

- **Associative:** \( (A \cup B) \cup C = A \cup (B \cup C) \)
- **Commutative:** \( A \cup B = B \cup A \)
- **Distributive:** \( A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \)
- **Identity:** \( A \cup \emptyset = A \)
- **Complement:** \( A \cup A' = U \) or \( A \cap A' = \emptyset \)
- **Double Complement:** \( (A')' = A \)
- **Idempotent:** \( A \cup A = A \)
- **Universal Bounds:** \( A \cup U = U \)
- **De Morgan’s Laws:** \( \overline{A \cup B} = \overline{A} \cap \overline{B} \) or \( (A \cup B)' = A' \cap B' \)

What are the Dual Identities?

Proving Set Identities

To prove \( X = Y \) prove \( X \subseteq Y \) and \( Y \subseteq X \)

How would you prove

- **Distributive Law:** \( A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \)
- **De Morgan’s Law:** \( \overline{A \cup B} = \overline{A} \cap \overline{B} \)

Proving conditional statements; e.g

- **Prove:** If \( A \subseteq B \) then \( A \cap B = A \)
- **Prove:** If \( A \subseteq B \) then \( A \cup B = B \)
- **Prove:** If \( A \subseteq B \) and \( A \subseteq C \) then \( A \subseteq B \cap C \)

The Empty Set: \( \emptyset \) is false

Prove: For all sets A, \( A \cap \emptyset = \emptyset \)
Proof by contradiction
Assume \( A \cap \emptyset \) is not empty
Therefore there is an element \( x \in A \cap \emptyset \)
\[
\begin{align*}
  x & \in A \\
  x & \in \emptyset
\end{align*}
\]
\( x \in \emptyset \) Contradiction!
Written Homework – #22 Due M 10/28/13

From Exercise Set 6.2 (p. 364)
#3, #4, #13, #16, #23, #24, #30