Math 171 – Discrete Mathematical Structures
Ch. 8.3 & 8.5

Recall: An relation R on a set A is an equivalence relation iff R is reflexive, symmetric, and transitive

Examples of equivalence relations
= (equality)
congruence modulo p
rational numbers
partitioning people by last names
equivalent Boolean expressions

Equivalence Relations
standard examples
Equivalence Relations and Set Partitions
Anti-symmetry
Partial Ordered Sets

Partitions, Equivalence Relations, & Equivalence Classes
Partition of a set A: A set of subsets \( A_1, A_2, \ldots, A_k \) where \( A_i \subseteq A \) for \( k = 1, 2, \ldots, n \) such that \( \bigcup_{i=1}^{k} A_i = A \) (i.e., their total union equals A) and \( A_i \cap A_j = \emptyset \) for \( i \neq j \) (i.e., the sets are mutually disjoint).

Theorem: Every partition on a set defines an equivalence relation
That is \( x \sim y \) iff \( x \in A_i \) and \( y \in A_i \)

Theorem: Every equivalence relation defines a partition on a set.

Equivalence Class: \([a] = \{x | x R a\}\) for some equivalence relation \( R \)

Anti-symmetry and Partial Orderings
Definition of anti-symmetry (vs. asymmetry): If \( \sim \) is a relation on a set A is anti-symmetric iff if \( a \sim b \) and \( b \sim a \) then \( a = b \)

Standard examples of anti-symmetric relations
| (divides) on Z
set containment \( \subseteq \)
lexicographic ordering

A relation \( R \) (denoted \( \preceq \)) on a set A is a partial ordering (aka poset) iff \( R \) (or \( \preceq \)) is reflexive, anti-symmetric, and transitive “\( \preceq \)” is the generic “partial ordering operator”

Representing Partially Ordered Sets
Hasse Diagrams – remove loops & transitive shortcuts
remove arrows & orient bottom to top

Comparables vs. non-comparables
Two elements \( a, b \) are comparable iff \( a \preceq b \) or \( b \preceq a \); otherwise they are non-comparable.

A poset is totally ordered (totally ordered set) iff all elements are comparable; otherwise it’s a partially ordered set.

Chain: a subset \( \{a\} \) of totally ordered elements in a poset

Maximal and Greatest, Minimal and Least elements.
In a poset A an element a is maximal iff for all elements b in A either \( b \preceq a \) or a and b are non-comparable. An element a is a greatest element iff \( b \preceq a \) for all b in A.

In a poset A an element a is minimal iff for all elements b in A either \( a \preceq b \) or a and b are non-comparable. An element a is a least element iff \( a \preceq b \) for all b in A.

Compatible Partial Orderings: Given two partial orderings \( \preceq \) and \( \preceq' \) on a set A, \( \preceq' \) is compatible with \( \preceq \) if and only if for all a and b in A if \( a \preceq b \) then \( a \preceq' b \).

Topological Sort: a partial ordering \( \preceq' \) which is compatible with \( \preceq \)
Written Homework #26 – Due M 11/11/13

Exercise Set #8.5 (p. 513)
    #1, #16, #22, #23