Math 171 – Discrete Mathematical Structures
Today’s Overview

Pigeonhole Principle

Counting Subsets – Combinations (Ch. 9.5)

Pigeonhole Principle
If \( n \) objects are distributed to \( m \) bins where \( n > m \) then one bin must contain at least two objects.

A function from one finite set to a smaller finite set cannot be one-to-one. There must be at least two elements in the domain that have the same image on the co-domain.

Pigeonhole Principle Examples
Given 13 people, at least 2 of them were born in the same month.

Given a drawer with 10 black socks and 10 white socks, you need to pull out 3 socks at random to get a matched pair.

Given the set of integers \( S = \{1,2,3,4,5,6,7,8\} \) if you randomly select 5 integers (w/o replacement) then at least one pair must sum to 9.

The decimal expansion of any rational number either terminates or repeats (use quotient-remainder theorem)

Generalized Pigeonhole Principle
If \( n \) objects are distributed to \( m \) bins where \( n > m \) and \( k = \frac{n}{m} \) then one bin must contain at least \( k+1 \) objects.

For any function \( f: X \rightarrow Y \) where \( |X| = n \), \( |Y| = m \), and \( n > m \), then for any positive integer \( k < \frac{n}{m} \), there is some \( y \in Y \) such that \( y \) is the image of at least \( k+1 \) distinct elements of \( X \).

Example: How many people are needed to guarantee that 3 people share the same birth month?

The Question: Given \( n \) objects how many ways can you choose a subset of size \( r \) (\( r \leq n \))? Denote this by \( C(n,r) \)

Aside: We introduced \( \binom{n}{r} = \frac{n!}{r!(n-r)!} \) or “\( n \) chose \( r \)” to compute the number of ways to choose subsets of size \( r \) from \( n \) objects.

Example: Given \( S = \{a,b,c,d,e\} \) enumerate all subsets of size 3 and determine \( C(5,3) \)

Permutations \( P(n,r) \) vs. Combinations \( C(n,r) \); the former are ordered.

Deriving \( C(n,r) = \binom{n}{r} = \frac{n!}{r!(n-r)!} \)

Using permutations: \( P(n,r) \) is the number of ways to arrange \( r \) objects chosen from a set of size \( n \). Claim \( P(n,r) = C(n,r) \times r! \)

\[
P(n,r) = C(n,r) \times r! \\
\frac{n!}{(n-r)!} = C(n,r) \times r! \\
\frac{n!}{r!(n-r)!} \times r = C(n,r)
\]
Examples

Given 8 people how many subcommittees of size 3 can be formed?

Poker Hands: 2 pairs
1. Chose 2 values for the pairs
2. Chose 2 cards for large value
3. Chose 2 cards for smaller value
4. Choose remaining card

Written Homework #32 – Due M 11/25/13

Exercise Set 9.4 (page 563)
#1, #7, #12, #14, #17, #24

Exercise Set 9.5 (page 581)
#2, #6, #8