Math 171 – Discrete Mathematical Structures
Today’s Overview
Walks, Trails, Paths, Circuits (review)
Connectedness
Euler Paths
Hamiltonian Circuits

Königsberg Bridge Problem
Is it possible for a person to take a walk around the town starting and ending at the same location and crossing each bridge exactly once? (Euler Circuit)

Walks, Trails, Paths, Circuits

Walk: alternating sequence of adjacent vertices and edges
Trail from v to w: a walk that does not contain a repeating edge
Path from v to w: a trail that does not contain a repeated vertex
Closed Walk: walk that starts and ends at the same vertex
Circuit: closed walk that contains at least one edge and does not contain a repeated edge (trail)
Simple Circuit: circuit that does not have any other repeated vertex (path) except the first and last.

Connectedness
Give a graph G, two vertices v and w are connected if there is a walk (trail, path) from v to w. A graph G is connected if given any two vertices v and w in G, there is a walk (trail, path) from v to w.

Connectivity Results: Let G be a graph
1. If G is connected then any two vertices can be connected by a path.
2. If vertices v and w are part of a circuit and one edge is removed, then there still exists a walk (trail, path) from v to w.
3. If G is connected and contains a circuit, then removing a edge from the circuit leaves G connected.

Euler Circuits
(a.k.a. Traveling Highway Inspector Problem)
An Euler Circuit for a graph G is a circuit that contains every vertex and every edge.

Theorem: A graph G has an Euler Circuit if every vertex has a positive even degree.
Contrapositive: If a graph G has a vertex of odd degree, then G does not have an Euler Circuit.
Theorem: If a graph G is connected and the degree of every vertex is even, then G has an Euler Circuit.

Hamiltonian Circuits
(a.k.a. Traveling Salesman Problem)
A Hamiltonian Circuit for a graph G is a simple circuit that includes every vertex of G.

Hamiltonian Circuit Results: If a graph G has a Hamiltonian Circuit, then G has a sub-graph H with the following properties:
1. H contains every vertex of G
2. H is connected
3. H has the same number of edges as vertices
4. Every vertex of H has degree 2.