Math 171 – Discrete Mathematical Structures
Review I

1. Sets and Set Containment: \(3 \in \{1, \{2\}, 3, \{2, 3\}\}\)?

2. Cartesian Products and Functions

3. Given the conditional statement: “If a positive integer \(n\) is prime then \(n\) equals 2 or \(n\) is odd”,
   a. write its negation
   b. write its converse
   c. write its inverse
   d. write its contrapositive

4. From a set of premises, use the standard argument forms to deduce the conclusion. Justify each step identifying it either as a premise or a rule of inference.
   1. \(p \lor q\)
   2. \(p \rightarrow r\)
   3. \(\sim r\)
   4. \(\therefore q\)

5. Determining valid/invalid argument from truth tables
   \(q \rightarrow r\)
   \(\sim r \rightarrow \sim p\)
   \(\therefore p \lor q \rightarrow r\)

6. Quantified Expressions – Rewrite the following expressions in symbolic form using the given predicates then negate the symbolic forms.
   a. All cars are red (using predicates \(\text{car}(\ )\), and \(\text{red}(\ )\))
   b. Some horses are white (using predicates \(\text{horse}(\ )\) and \(\text{white}(\ )\))
   c. All positive integers have a prime that divides it (using predicates \(\text{positive_integer}(\ ), \text{prime}(\ ), \text{divides}(\ , \ )\))
   d. There is a food that everyone likes (using \(\text{food}(\ ), \text{person}(\ ), \text{likes}(\ , \ )\))

7. Some of the following arguments may be valid by universal modus ponens or universal modus tollens; other may be invalid by exhibiting an inverse error or a converse error. State which are valid or invalid and justify your answer.
   a. All cows eat grass
      Bessie does not eat grass
      Bessie is not a cow
   b. Every good boy does fine
      Lynn is not a boy
      Lynn does not do fine.
   c. All girls are smart
      Alex is smart
      Alex is a girl
   d. All math courses are fun
      Math 171 is a math course
      Math 171 is fun.

8. Determine whether the following statement is true or false. Justify your answer with a proof or a counter example, as appropriate. Use proper definitions of terms. You may assume the standard rules of algebra and the assumption that the set of integers is closed under addition, subtraction and multiplication.
   The product of any two odd integers is odd.

9. Using Truth Tables determine which of the following statements is a tautology (if either). Show all work to receive credit
   a. \((p \rightarrow q) \rightarrow p\)
   b. \(p \rightarrow (q \rightarrow p)\)
   X. Rewrite \(p \rightarrow q\) using only \(\sim\) and \(\land\)

Test #2

1. Complete the statement of the Quotient Remainder Theorem
   Apply it to case where \(n = -17\) and \(d = 3\)

2. Sequences and Series

3. Given recurrence relation \(g_1 = \begin{cases} 1 \text{ if } k = 1 \\ 2g_{k-1} \text{ if } k > 1 \end{cases}\)
   find the values of the first four terms & guess an explicit formula for the sequence
4. Prove: if \( x \) is even then \( x^2 \) is even

True or False: For all integers \( x \) if \( x^2 \) is odd then \( x \) is odd. If false give a counter-example, if true how would you prove it?

5. Mathematical Induction Proof: \( \sum_{k=1}^{n} k = \frac{n(n+1)}{2} \)

6. Set Theory Proofs: Prove if \( A \subseteq B \) then \( A \cap C \subseteq B \cap C \)

7. Venn Diagrams

8. True or False

X. Prove algebraically using set identities \( A \cup (B - A) = A \cup B \)