“Let no man ignorant of geometry enter here”
motto across archway of Plato’s Academy

Historical Overview
490 BCE – Battle of Marathon; 480 BCE - Battle of Thermopylae;
431 – 404 BCE Peloponnesian War

Plato 427 – 347 BCE; Eudoxus 408 – 355 BCE;
Aristotle 384 – 322 BCE

Alexander the Great: 356 – 323 BCE
Alexandra founded – ca. 332 BCE
Euclid’s Elements - 300 BCE

Eudoxus 408 – 355 BCE

- Theory of Proportions
  fixed logical scandal of incommensurate quantities
  Two quantities a and b are commensurate iff there is a 3rd quantity c such that a and b are integral multiples of c.

- Method of Exhaustion (proto-integration)
  used to compute areas and volumes of non-rectilinear figures

Axiomatic Method

- Begin with handful of (self-evident) axioms/postulates and definitions
- Logically develop proofs of propositions based on axioms, definitions and previously proved propositions
- Logical validity of propositions
- Avoids circular arguments
- Economy of presupposition

Ch 2: Euclid’s Proof of the Pythagorean Theorem

Book I Preliminaries
Definitions, Postulates, Common Notions, The 5th Postulate
The Early Propositions
I.1, I.2, I.5, I.16
Enter Parallelism
I.27, I.29, I.32
Triangles and Parallelograms
I.31, I.35, I.37, I.41
The Proof of I.47

The Five Postulates

1. [It is possible] to draw a straight line from any point to any point
2. [It is possible] to produce a finite straight line continuously in a straight line
3. [It is possible] to describe a circle with any center and distance (i.e. radius)
4. All right angles are equal to one another
5. If a straight line meets two straight lines so as to make two interior angles on the same side of it taken together less than two right angles, the lines if extended shall meet on that side on which the angles are less than two right angles
Early Propositions

I.1 On a given finite straight line, to construct an equilateral triangle
I.2 From a given point (A) to draw a straight line equal to another given straight line (BC)
I.3 From the greater of two given straight lines cut off a part equal to the lesser
I.4 SAS congruency
I.5 The angles at the base of an isosceles triangle are equal to one another and if the equal sides be produced, the angles on the other side of the base shall be equal to one another

Early Propositions

I.15 Vertical angles are equal
I.16 In any triangle if one of the sides is produced, then the exterior angle is greater than either of the interior or opposite angles (used for I.26 AAS congruence)

Parallelism

I.27 If a straight line falling on two straight lines makes alternate angles equal to one another, then the straight lines are parallel to each other
I.29 A straight line falling on parallel straight lines makes the alternate angles equal to one another, the exterior angles equal to the interior and opposite angle, and the sum of the interior angles on the same side equal to two right angles.

Parallelism

I.32 In any triangle if one of the sides is produced than the exterior angle equals the sum of the two interior and opposite angles

Triangles and Parallelograms

I.31 To draw a straight line through a given point parallel to a given straight line.
I.35 Parallelograms which are on the same base and in the same parallels equal one another
I.37 Triangles which are on the same base and in the same parallels equal one another.
I.41 If a parallelogram has the same base with a triangle and is in the same parallels, then the parallelogram is double the triangle.
Proof of I.47

I.46  To describe a square on a given straight line.

I.47  In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.

I.48  If in a triangle the square on one of the sides equals the sum of the squares on the remaining two sides of the triangle, then the angle contained by the remaining two sides of the triangle is right.

Again the 5th Postulate

Proclus’ axiom: If a line intersects one of two parallel lines it must intersect the other also
Equidistance postulate: Parallel lines are everywhere equidistant
Playfair’s postulate: Through a point not on a given line there can be drawn one and only one line parallel to the given line
The triangle postulate: The sum of the angles of a triangle is two right angles

The 5th Postulate

Giovanni Girolamo Saccheri (1667-1733)
Saccheri Rectangle
Carl Fredrick Gauss (1777-1855)
Johann Bolyai (1802-1860)
Nikolai Lobachevski (1793-1856)
Georg Friedrich Bernhard Riemann (1826-1866)
Eugenio Beltrami (1835-1900)
Non-Euclidian Geometry: AAA is congruence relation