Hippocrates (of Chios) Quadrature of the Lune
c.a. 440 B.C.E.

Volume of a Frustum

"If you are told: A truncated pyramid of 6 for the vertical height by 4 on the base by 2 on the top. You are to square this 4, result 16. You are to double 4, result 8. You are to square 2 , result 4. You are to add the 16, the 8, and the 4, result 28. You are to take a third of 6, result 2. You are to take 28 twice, result 56. See it is 56. You will find it right" - Moscow Papyrus ca. 1850 BCE

Using the above find the volume of a truncated square pyramid with 10 for the vertical height by 7 on the base by 3 on the top.

Translate the description for the volume of a frustum into an algebraic formula.

An Overview of Greek Mathematics

Demonstrative Mathematics: “a theoretical, deductive system in which emphasis was placed upon proving critical relationships” – Dunham p. 5

Thales (ca. 640 – 546 BCE)
proved (?): vertical angles equal angle sum of triangle equals two right angles base angles of isosceles triangle equal angle inscribed in semicircle is right angle

Quadrature

“The quadrature (or squaring) of a plane figure is the construction – using only compass and straightedge – of a square having the area equal to that of the original plane figure.”

Every rectilinear figure is quadrable.

Quadrature of a Rectangle

Needed:
Pythagorean theorem

Quadrature of a triangle and of any other rectilinear figure
Hippocrates’ Quadrature of the Lune

Needed:
Pythagorean Theorem.
Angle inscribed in a semi-circle is right.
Areas of two semi-circles are to each other as the squares of their diameters.

Three Classic Problems

Squaring the Circle
Doubling the Cube (Delian Problem)
Trisecting an Angle

Problems were planar, solid, or linear
Planar – problems solved using compass & straight-edge only
Solid – could use curves derived from conic sections
Linear – could use more complicated curves (e.g. spirals – or quadratrix of Hippias)

Quadratrix of Hippias

Epilog

What are constructable numbers?
Can you square the circle?
Who was Ferdinand Lindemann?
How many lunes are quadrable?
Leonard Euler & Lunes – What the connection?
Who are N.G. Tschebatorew and A.W. Dorodnow

What do you remember from your high school geometry?