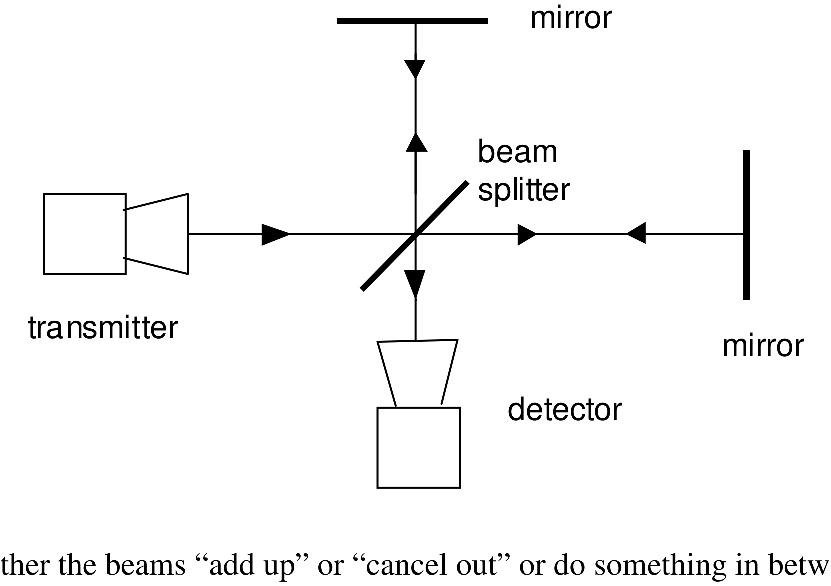
Physics 220, Lab 1, Spring 2009
The Microwave Michelson Interferometer

The Michelson interferometer (see Sec. 1.4 and 1.5 in TZD) is famous for its use in the Michelson-Morley experiment. This experiment showed (to the surprise of Michelson and Morley themselves) that the speed of light is the same in all directions in an inertial reference frame (which came to be one of the postulates of special relativity). But the Michelson interferometer has many other uses, such as measuring small distances. This week, we will use a Michelson interferometer to measure the wavelength of microwaves produced by a microwave transmitter. Later in the semester we will do a Bragg scattering experiment with microwaves using the same transmitters, for which we will need to know the wavelength.

Safety: The intensity of our microwave transmitters is well within standard safety levels, but it is good practice to avoid unnecessary or close-range exposure (for example, don't stand in the microwave beam or look into the transmitter at close range when it's on). ***Pacemakers or other electronic medical devices may be affected by the microwave radiation. Please let me know if this is an issue.***

Background:
Microwave radiation from the source is divided into two beams by the partial reflector or "beam splitter" (for microwaves, a sheet of masonite works well for this purpose). The transmitted beam travels to one "mirror" (a flat metal plate) and the reflected beam travels to another mirror. These two beams then return to the partial reflector, and a portion of each travels to the detector, as shown.



Whether the beams "add up" or "cancel out" or do something in between when they recombine in going to the detector is determined by the total difference in distance the two beams travel from the point at which they split to the point at which they recombine. If the two beams travel the same distance, a "crest" of one wave train will arrive at the same time as a crest of the other wave train, and they will add up (constructive interference). In this case, the detector will indicate a large reading. If the difference in distance is a multiple of a wavelength of the radiation (λ, 2λ, 3λ, ...), the two waves will also arrive in phase and add up.

If the difference in distance is equal to λ/2, 3λ/2, ... a crest of one wave will arrive at the same time as a trough in the other wave, and the two will tend to cancel (destructive interference). This will produce a low reading on the meter of the detector. (Stray radiation and the tendency of one beam to be stronger than the other usually keep the deflection of the meter from being exactly zero.)

Since each beam travels twice between the beam splitter and the mirror, moving one mirror a distance equal to λ/4 will change the path length by λ/2 and therefore should change constructive interference (a maximum) into destructive interference (a minimum) and vice versa. Thus, by measuring how far the mirror must move to change a maximum into a minimum, the wavelength of the radiation can be determined.

Prelab questions (work out in your lab book):

1. The microwave transmitters we're going to use in this lab are labeled "15 mW, 10.5 GHz."

a) Calculate the expected wavelength of the wave they produce. Verify (inside back cover of TZD) that this wavelength really does correspond to the microwave region of the electromagnetic spectrum.

b) Given your answer to a) and the information in the lab writeup, how far will you have to move one mirror of the Michelson interferometer to measure 20 alternating minima and maxima? Explain.

c) What do you expect the period of the wave produced by the transmitter to be? Do you think you'll be able to observe the time variations of the waves?

d) What does the 15 mW tell you about the transmitters?

2. Will two waves interfering destructively always result in zero amplitude? (Hint: what if the two waves don't have equal amplitudes?)

Procedure:

Getting to know the apparatus (with the help of the documentation as needed):

Set up the microwave apparatus, examining it as you do so. What do the T and R marks on the mounting stands represent?

Plug in the transmitter and turn on the receiver. Have them face each other and see what effect rotating the transmitter horn relative to the receiver horn has. For best sensitivity, should the horns be aligned or at an angle to each other?

Adjust the Intensity dial on the receiver to see what effect it has. Move the transmitter and receiver closer to and then farther from each other and note what happens to the readings. What does the "10x" setting mean?

What does the Variable Sensitivity knob do? If recording transmitter readings is important, what happens if you change the Variable Sensitivity knob during an experiment?

Set up the Michelson Interferometer as shown. Place one of the mirrors at about the middle of its track and the other mirror about 40 cm from the beam splitter, which should be set at about 45°. Adjust the angle of the first mirror to obtain a maximum reading on the detector meter. (If the needle goes off scale, or goes below half scale, change scales.) Then move the second mirror slowly away from the beam splitter until the meter reading is a minimum. Rotate the beam splitter slightly to see whether you can reduce the meter reading still further. Make a note of the scale; if you need to change the scale during an experiment, be sure to note that in your lab notebook. Move the receiver back and forth and notice the response of the meter.

Move the second mirror slowly away from the beam splitter, determining with as much precision as possible the positions of the mirror at which maxima and minima occur. Record at least 20 mirror positions for alternate maxima and minima, $x_1, x_2, x_3, \ldots, x_{20}$, where the odd subscripts are maxima and the even are minima, or vice versa.

Data analysis:

At first, you might be tempted to simply average the successive differences in order to determine the best value for $\lambda/4$, that is, to find $\frac{(x_{20}-x_{19})+(x_{19}-x_{18})+\ldots+(x_2-x_1)}{19}$.

This turns out to be a poor way to process all that data you took, though, because the above expression reduces to $(x_{20}-x_1)/19$—which means you're throwing away all but your first and last values! A much better procedure is to find the average of the following set of differences, in which each reading is used only once: $\frac{x_{20}-x_{10}}{10}, \frac{x_{19}-x_9}{10}, \ldots\ldots, \frac{x_{11}-x_1}{10}$.

The usual way to estimate the uncertainty of a value obtained by averaging experimental values is by calculating the standard deviation of the mean (SDOM), which is the standard deviation divided by the square root of the number of points.

Find the wavelength of the microwave radiation and determine the uncertainty in its value. Compare to the manufacturer's value (found in the prelab).

As an alternative (and much more visual) way to find the wavelength, make a graph of the points $(1,x_1)$, $(2,x_2)$, $(3,x_3)$,...,$(20,x_{20})$ and determine the slope and the uncertainty in the slope. (This can be done in Excel—see the handout on using the LINEST function. It can also be done in LoggerPro or in a variety of other software packages.) Then use the slope to determine the wavelength and its uncertainty. Compare the graphical method to the averaging method.

Post-lab assignment:

This may be done in groups or individually. Send me a *Mathematica* file with the results by the beginning of lab next week.

The fundamental principle of this lab is the superposition of waves. In each of the experiments, a wave is split into two or more segments and recombined after each segment has traversed paths of different lengths. At a given time t=0 the recombined wave can be expressed as:

$$y_T = y_1 + y_2 = A_1 \sin\left(\frac{2\pi x}{\lambda}\right) + A_2 \sin\left(\frac{2\pi(x+\Delta x)}{\lambda}\right)$$

where λ is the wavelength, A_1 and A_2 are the amplitudes of the two waves, x is the distance traveled by both waves, and Δx is the extra distance traveled by the second wave relative to the first wave.

a) Consider microwaves with a wavelength of 4 cm. Using *Mathematica*, plot y_T as a function of x for specific values of Δx, taking A_1 and A_2 to be equal. Let Δx have both negative and positive values. Choose a reasonable value for the range of x (i.e. show at least one whole wave) and a reasonable range to vary Δx over (i.e. show constructive, destructive, and "in between" interference). You could use the Manipulate command to vary Δx on a single graph. Hint: use PlotRange in the argument of the Plot function to fix the x and y scales so the graph doesn't resize itself as you change Δx.

b) Now assign A_1 and A_2 different values and redo 1) and 2). What do you now observe about the maxima and minima from these plots? Relate this to what you saw when doing the microwave Michelson experiment.