

# Physics 220 Lab 2, Spring 2009

## The Charge to Mass Ratio of the Electron

### Background:

J.J. Thomson won the Nobel Prize for measuring the charge-to-mass ratio of the electron (thus establishing that atoms contain particles known as electrons). He used a crossed magnetic and electric field to determine the speed of the electrons, and then measured their deflection in an electric field to determine  $e/m$ . We will do something similar, except that we will determine the speed by accelerating the electrons through a known potential difference, and will measure their deflection in a magnetic field.

The force,  $\mathbf{F}$ , on a particle of charge  $q$  moving with velocity  $\mathbf{v}$  in a magnetic field,  $\mathbf{B}$ , is given by:

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B}) \quad (1)$$

where the direction of  $\mathbf{F}$  is given by the cross product.

If  $\mathbf{B}$  is uniform, and if  $\mathbf{v}$  is perpendicular to  $\mathbf{B}$ ,  $\mathbf{F}$  is always perpendicular to  $\mathbf{v}$  and consequently can only change the direction of  $\mathbf{v}$  and not its magnitude. This means the motion of the charged particle is circular, and the acceleration is purely centripetal. We have then according to Newton's second law:

$$F = ma = m \frac{v^2}{r} \quad (2)$$

In this experiment, giving the electrons a well-defined speed is accomplished by accelerating them through an electric potential,  $V$ . Equating the potential energy before the acceleration to the kinetic energy after the acceleration yields

$$qV = \frac{1}{2}mv^2 \quad (3)$$

Combining equations (1), (2), and (3) and solving for the charge to mass ratio of the electron yields:

$$\frac{q}{m} = \frac{2V}{B^2 r^2} \quad (4)$$

where  $q$  is the charge on the electron (usually called  $e$ ) and  $m$  is its mass.

To determine a numerical value for  $e/m$ , we need to know the magnetic field  $B$ . We can measure  $B$  directly, or we can calculate it based on the configuration of current-carrying coils that produces it. For this experiment, two circular coils (called Helmholtz coils) are used. For this configuration, the magnetic field is nearly uniform in the region between the coils and is directly proportional to the current in the coils. The equation for  $B$  as a function of the current  $I$  is (see, for example, Serway):

$$B = \frac{\mu_0 NI}{(5/4)^{3/2} a} \quad (5)$$

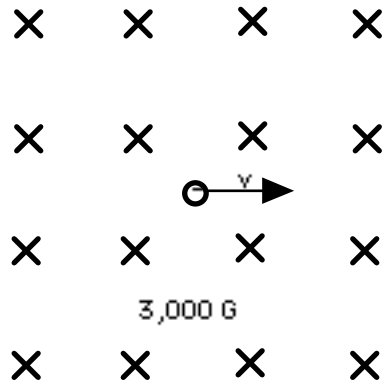
where  $a$  is the radius of the Helmholtz coils,  $N$  is the number of turns, and  $\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}$ . The working equation for this experiment is found by substituting equation (5) into equation (4).

$$\frac{e}{m} = \frac{2V(5/4)^3 a^2}{(\mu_0 N I r)^2} \quad (6)$$

From equation (6), the slope of the line of  $r$  vs.  $1/I$  will depend on  $e/m$ . This suggests that one way to do the experiment is to graph  $r$  (on the y axis) vs.  $1/I$  (on the x axis), and find the slope. This method has the advantage of not only getting a numerical value for  $e/m$ , but also of checking that the functional relationship between  $r$  and  $I$  is correct.

### Pre-lab Questions (please work these out in your lab book)

- Please work out the substitutions to derive equation (6). Then modify equation (6) so that  $r$  is written as a function of  $(1/I)$ . Identify the slope of this equation.
- An electron has an energy of 200 eV (that'll be typical in this lab, as we'll be accelerating electrons through potential differences of a few hundred volts).
  - What is the electron's speed, using the full relativistic formulas (see TZD pp. 56-8, especially Example 2.5)?
  - What is the electron's speed, using the classical (nonrelativistic) expression for kinetic energy?
  - What is the percent difference between a) and b)? Is using the nonrelativistic expression probably OK in this experiment, and why? (Hint: think about the level of precision you're likely to get from the measurements.)
- An electron enters a uniform magnetic  $\mathbf{B}$  field as shown. At the instant shown, what is the direction of the force,  $\mathbf{F}$ , acting on the electron?



**PROCEDURE:** Detailed instructions for this experiment will depend upon the apparatus you are assigned. Specifications for each experimental apparatus are kept with the apparatus. **Before you begin, study the apparatus and explain the function of each element to your instructor. There are parameters within which the experiment must be done. Know them.**

When you have mastered the apparatus, you may begin the experiment. For a particular accelerating voltage, measure five different radii. It might be helpful to take these values several times to gain information on their uncertainties. Then repeat for a different accelerating voltage.

### Embedded Questions

Please answer these questions using the experience gained from gathering your data.

1. What are the uncertainties in the current and voltage measurements? How could you tell if there were systematic errors in the meters used in the experiment? Which measurements are the most critical?
2. Suppose the experiment were done by keeping the magnetic field current constant and varying only the accelerating voltage. If the beam were aligned with the outermost post, how would you have to adjust the accelerating voltage to cause the beam to be aligned with the inner post? Please explain the physics behind your answer.
3. It is not necessary to record the cathode current (that is, the current associated with the accelerating voltage) to determine the ratio of  $e/m$ . Why? Why is it still a good idea to monitor it?
4. The electron beam has a finite size both horizontally and vertically. This suggests that there is some uncertainty associated with the data. Estimate this uncertainty and indicate how it affects your values for  $e/m$ .
5. For the largest and smallest magnetic field currents you used, calculate the magnitudes of the associated magnetic fields. Compare to the size of the Earth's magnetic field. Were we justified in neglecting the contribution of the Earth's field? Explain.
6. Compare the smallest magnetic force you used to the weight of the electron. Is the gravitational force on the electron a significant effect? Explain.

**Please check with your instructor before proceeding.**

ANALYSIS: Plot  $r$  vs  $1/I$  for the two trials (i.e.  $r$  is the dependent variable,  $1/I$  the independent variable). The two data sets can be placed on a single graph. Determine the uncertainty in the current and the radius and include the associated error bars on the data points.

Fit a line to your data to find the slopes of your two curves, with uncertainty. How good is the linear fit?

From the slope of the data, calculate the value of  $e/m$  and its uncertainty. Is there agreement with the accepted value within your uncertainties?

### Summary Questions to consider

Make some comments about the accuracy (closeness to the accepted value) and precision (size of uncertainty) of your results.

If you wanted to improve the accuracy of the experiment, what changes would you make? If you wanted to improve the precision (i.e. reduce the size of the error bars), what changes would you make?