# Physics 220, Lab 10, Spring 2009
# Evanescent waves and tunneling

## I. INTRODUCTION

According to quantum mechanics, even in cases in which a particle does not have enough energy to get over a barrier, there is some probability that the particle can pass to the other side of the barrier (“tunneling”). However, our classically conditioned physical intuition finds it difficult to understand how any of the particle can get to the right of the barrier when they must pass through a region in which they are energetically forbidden. The ability to "tunnel" through a forbidden region is a characteristic of waves. Such waves are called evanescent waves when encountered in optics. Here we’ll use microwaves to investigate evanescent waves and the related phenomenon of frustrated total internal reflection, which is analogous to the phenomenon of tunneling in quantum mechanics.

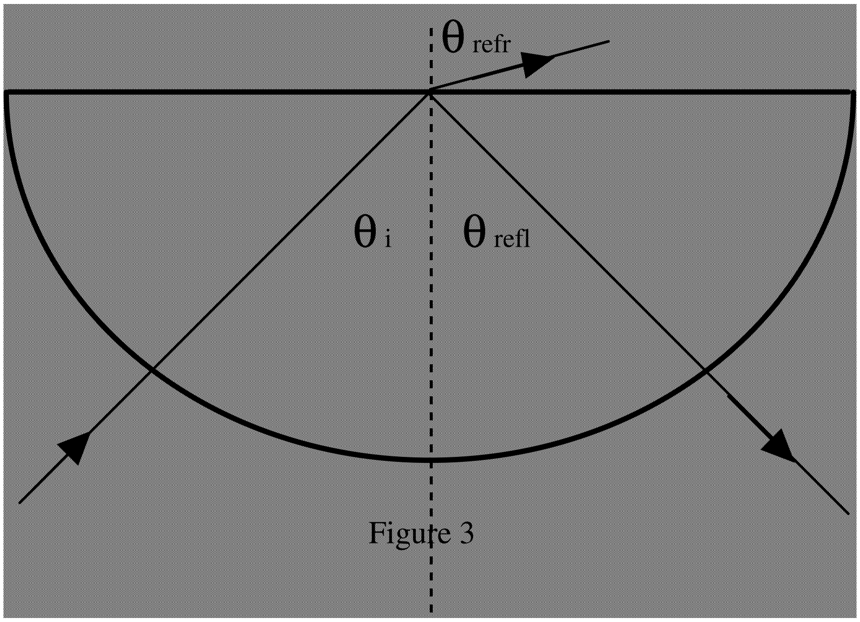


Figure 3

Suppose we have electromagnetic waves (light, x-rays, microwaves) incident on the semi-circular region as shown in the figure above. This region has an index of refraction, n. The medium surrounding this region is assumed to have an index of refraction equal to one.

At the straight surface, part of the incident wave will be reflected and part transmitted. From our study of geometrical optics, we have learned that:

$$\theta_i = \theta_{refl} \qquad (1)$$

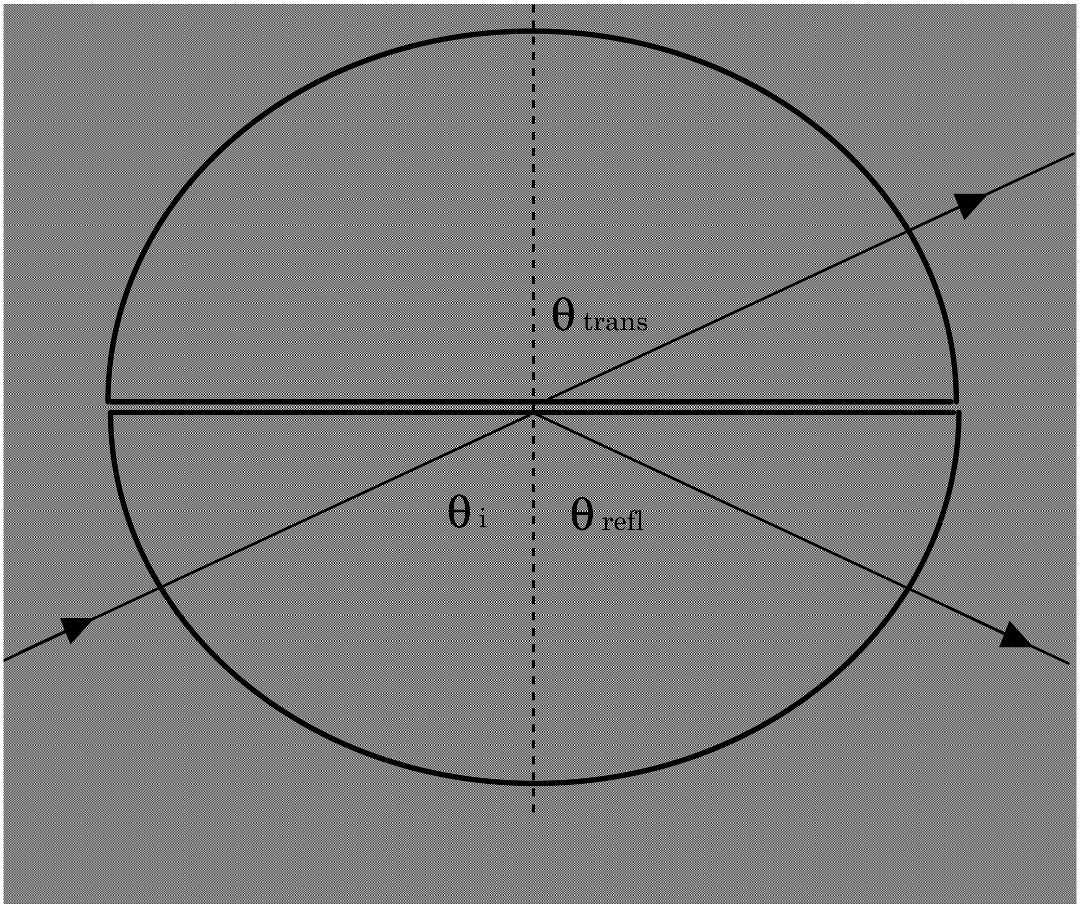
and by Snell’s law:

$$n \sin\theta_i = (1) \sin\theta_{refr} \qquad (2)$$

Since n is greater than one, we expect $\theta_{refr}$ to be greater than $\theta_i$ as is indicated on the diagram. As $\theta_i$ increases from 0, $\theta_{refr}$ increases faster until it equals $90^o$. At this angle, none of the electromagnetic radiation can be transmitted, and all of it is reflected from the surface. This value of $\theta_i$ is known as the critical angle, $\theta_c$, and it can be calculated using (2):

$$\sin\theta_C = \frac{1}{n} \qquad (3)$$

As $\theta_i$ is increased beyond $\theta_C$, there can be no radiation transmitted into the "second" medium. Suppose a second semi-circular region is now placed near, but not touching, the first region as shown in the figure below.



Even though $\theta_i$ is greater than $\theta_C$, there will now be a transmitted wave. The wave has "tunneled" through a region where it was forbidden and appeared in the region beyond. This is the phenomenon we will investigate experimentally. In optics this is called <u>Frustrated Total Internal Reflection</u>, and it is due to the exponential falloff of the electric and magnetic fields of the EM wave in the gap.

**Embedded question:** A light ray in glass (n = 1.5) is incident on a glass/air surface.

a) If the angle of incidence is $30^o$ sketch the incident, reflected, and transmitted rays at their respective angles. (i.e. calculate the angles and then sketch the rays)

b) If the angle of incidence is $60^o$ sketch the incident, reflected, and transmitted rays at their respective angles. (i.e. calculate the angles and then sketch the rays)

c) Please comment on the difference between these situations.

d) Define (in words) the term "critical angle," and calculate its value for light going from glass to air.

**Embedded question:** Explain why semi-circular regions are useful for doing this experiment (rather than, say, rectangular blocks of material).

**Embedded question:** Based on the discussion above, and on what you know about tunneling in quantum mechanics, how do you expect the intensity transmitted through a gap between two prisms will decrease as the distance between the prisms is increased? (That is, what specific functional form do you expect the transmitted intensity as a function of distance to have?)

## II. PROCEDURE

**PART ONE:** The first part of the experiment will explore whether microwaves behave similarly to light waves in terms of reflection and refraction, and to measure their index of refraction in paraffin. You will use the PASCO microwave set that you used earlier for the microwave Bragg's Law experiment. You'll also have two semicircular (in 3D, semi-cylindrical) pieces of paraffin. Since you've used the microwave apparatus several times already, you should be somewhat familiar with it, and so the procedure you'll use here will be designed by you in consultation with me.

Specifically, develop a procedure that enables you to investigate the angles of the reflected and refracted waves as a function of the incident intensity and angle. The goal is to check that Snell's law and the law of reflection hold, and to measure the index of refraction of the paraffin at the wavelength of these microwaves. As you design your procedure, think about what parameters you expect to be able to extract from your data, and how your procedure will work to ensure accuracy and precision as well as to allow you to estimate your uncertainty. You might also look at the questions below to make sure that you'll be able to answer them using your data.

After you are satisfied that you have a reasonable procedure and that you understand how the resulting data should be analyzed, and what parameters can be found, discuss this with your instructor. When she is satisfied that you understand what you are going to do, you may proceed to gather and analyze data.

Based on your data, answer the following questions:

1. How does the reflected angle compare to the angle of incidence? How does the angle of refraction compare to the angle of incidence? Relate these observations (quantitatively) to the law of reflection and to Snell's Law.

2. What is the value of the index of refraction (with uncertainty) of the paraffin for electromagnetic waves of this wavelength?

**PART TWO:** You now are ready to investigate the tunneling as a function of the separation distance between the two semi-circular regions. Again, design a procedure that will achieve this. What data will you need to gather? How should it be analyzed? Please check with your instructor before you gather and analyze your data.

Use your data and analysis to answer the following questions:

1. How does the transmission intensity drop off with separation distance (exponentially, linearly)?

2. What is the effective “half thickness” of the gap, the thickness of the gap at which the intensity has dropped off to half its value? How does this thickness compare to the wavelength of the microwaves you are using?

3. What are the similarities and differences of this situation with microwaves to quantum mechanical tunneling?