5. TZD 7.29 normalization integral for particle in 1D rigid box

show: $\int_0^a \sin^2\left(\frac{n\pi x}{a}\right) dx = \frac{a}{2}$

Appendix B $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

so $\int_0^a \sin^2\left(\frac{n\pi x}{a}\right) dx = \int_0^a \left(\frac{1}{2} - \frac{1}{2}\cos\left(\frac{2n\pi x}{a}\right)\right) dx$

$= \frac{1}{2}\int_0^a dx - \frac{1}{2}\int_0^a \cos\left(\frac{2n\pi x}{a}\right) dx$ $\quad u = \frac{2n\pi}{a}x;\ du = \frac{2n\pi}{a}dx$

$x = a \rightarrow u = 2n\pi$

$= \frac{1}{2}x\Big|_0^a - \frac{1}{2}\left(\frac{a}{2n\pi}\right)\int_0^{2n\pi} \cos u\, du$

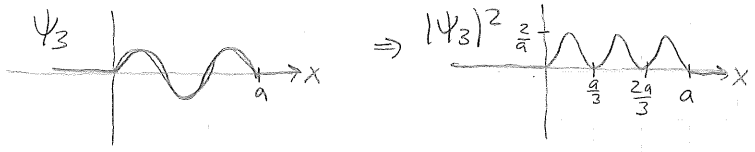
$= \frac{a}{2} - \frac{a}{4n\pi}\sin u\Big|_0^{2n\pi} = \frac{a}{2} - \frac{a}{4n\pi}(\sin 2n\pi - \sin 0) = \frac{a}{2}$ ✓

($\sin 2n\pi = 0$ for $n = 1, 2, 3, \ldots$; $\sin 0 = 0$)

for any $n = 1, 2, 3, \ldots$

6. TZD 7.30 1D rigid box, length a $\quad \psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right)$

a) $|\psi_3(x)|^2 = \frac{2}{a}\sin^2\left(\frac{3\pi x}{a}\right)$



b) most probable positions $x_{mp} = x$ where $|\psi_3|^2$ is greatest

$x_{mp} = \frac{a}{6}, \frac{a}{2}, \frac{5a}{6}$

c) probability of finding particle in $[0.50a, 0.51a]$

$P = \int_{0.5a}^{0.51a} |\psi_3|^2 dx = \int_{0.5a}^{0.51a} \frac{2}{a}\sin^2\left(\frac{3\pi x}{a}\right) dx$

since Δx is small $(0.01a)$, $P \approx |\psi_3(0.505a)|^2 \Delta x$

$= \frac{2}{a}\sin^2\left(\frac{3\pi(0.505a)}{a}\right)(0.01a) = 0.02$ (2%)

($\frac{3\pi(0.505a)}{a}$ radians)

probability in $[0.75a, 0.76a] \approx |\psi_3(0.755a)|^2 \Delta x$

$= \frac{2}{a}\sin^2\left(\frac{3\pi(0.755a)}{a}\right)(0.01a) = 0.01$ (1%)