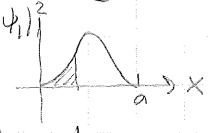
7. TZD 7.32 ground state ($n=1$) of 1D rigid box $\psi_1(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{\pi x}{a}\right)$

$$P\left(0 \le x \le \frac{a}{3}\right) = \int_0^{a/3} |\psi(x)|^2 dx$$



$$= \int_0^{a/3} \frac{2}{a}\sin^2\left(\frac{\pi x}{a}\right) dx = \frac{2}{a}\int_0^{a/3}\left(\frac{1}{2} - \frac{1}{2}\cos\left(\frac{2\pi x}{a}\right)\right) dx \qquad u = \frac{2\pi}{a}x$$

$$= \frac{1}{a}\int_0^{a/3} dx - \frac{1}{a}\frac{a}{2\pi}\int_0^{2\pi/3}\cos u\, du$$

$$= \frac{1}{a}(x)\Big|_0^{a/3} - \frac{1}{2\pi}(\sin u)\Big|_0^{2\pi/3}$$

$$= \frac{1}{3} - \frac{1}{2\pi}\left(\sin\frac{2\pi}{3} - \sin 0^{\nearrow 0}\right) = 0.1955 \quad \rightarrow \text{reasonable based on graph}$$

8. TZD 7.34 expectation value for x $\langle x\rangle = \int_{-\infty}^{\infty} x\,|\psi_n(x)|^2 dx$

and $\psi_3(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{3\pi x}{a}\right)$ $\quad 0 \le x \le a$

$$\Rightarrow \langle x\rangle = \int_0^a x\frac{2}{a}\sin^2\left(\frac{3\pi x}{a}\right) dx \quad \leftarrow \text{again, use } \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\langle x\rangle = \frac{2}{a}\int_0^a x\left(\frac{1}{2} - \frac{1}{2}\cos\left(\frac{6\pi x}{a}\right)\right) dx$$

$$= \frac{1}{a}\int_0^a x\,dx - \frac{1}{a}\int_0^a x\cos\left(\frac{6\pi x}{a}\right) dx \quad \leftarrow y = \frac{6\pi x}{a},\ dy = \frac{6\pi}{a}dx;\ x=a \Rightarrow y = 6\pi$$

$$= \frac{1}{a}\left(\frac{1}{2}x^2\right)\Big|_0^a - \frac{1}{a}\left(\frac{a}{6\pi}\right)\int_0^{6\pi} y\cos y\, dy \quad \leftarrow \text{integrate by parts } \begin{cases}\int u\,dv = uv - \int v\,du \\ u = y,\ dv = \cos y \\ \rightarrow v = \sin y\end{cases}$$

$$= \frac{a}{2} - \frac{1}{6\pi}\left(y\sin y\Big|_0^{6\pi} - \int_0^{6\pi}\sin y\, dy\right)$$

$$= \frac{a}{2} - \frac{1}{6\pi}\left[6\pi\sin 6\pi^{\nearrow 0} - 0) + \cos y\Big|_0^{6\pi}\right]$$

$$= \frac{a}{2} - \frac{1}{6\pi}\left[\cos 6\pi - \cos 0\right] = \frac{a}{2} - \frac{1}{6\pi}[1-1] = \frac{a}{2} \quad \text{as expected}$$