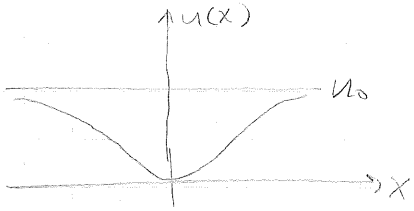
Physics 220 Assignment 6

1. TZD 7.37 $U(x) = U_0(1 - e^{x^2/a^2})$

a)



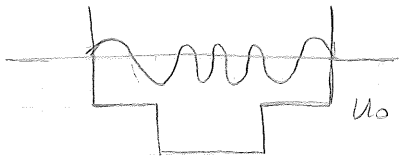
b) classical turning points $E = U(x) = U_0(1 - e^{-x^2/a^2})$

solve for x: $1 - \frac{E}{U_0} = e^{-x^2/a^2}$

$$x^2 = -a^2 \ln\left(1 - \frac{E}{U_0}\right) = -a^2 \ln\left(\frac{U_0 - E}{U_0}\right)$$

+ since $-\ln\eta = \ln\frac{1}{\eta}$, $x^2 = a^2 \ln\left(\frac{U_0}{U_0 - E}\right) \Rightarrow x = \pm a\left(\ln\left(\frac{U_0}{U_0 - E}\right)\right)^{1/2}$

2. a) TZD 7.41



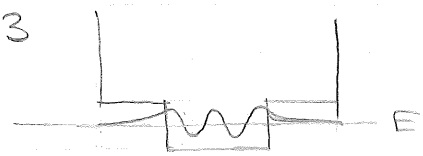
8th excited state ($n = 9$) for $E > U_0$

λ shorter where $K = E - U$ larger (center)

9 antinodes

$\psi = 0$ outside ($U \to \infty$)

b) TZD 7.43



now $0 < E < U_0$

4th excited state ($n = 5$)

falls off exponentially outside center (till it reaches $U \to \infty$, then zero)

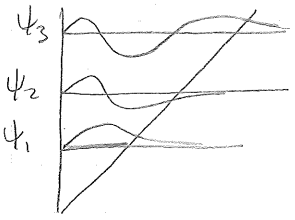
3. TZD 7.44

a) hard (rigid) surface at $x = 0$ (instantly turns around there)

for $x > 0$, $U \propto x$; uniform gravitational field has $U(y) = mgy \propto y$

b) ψ_n has n antinodes; $\psi = 0$ for $x \leq 0$, oscillates for $x > 0$ up to turning point, then decays exponentially as $x \to \infty$

curvature greatest where K greatest ($E - U$ greatest)



4. TZD 7.45 a) $E_n(\text{finite well}) \lessapprox E_n(\text{infinite well}) = n^2 \frac{\pi^2\hbar^2}{2ma^2}$

highest state of finite well $E_n \approx U_0$

so $n^2 \frac{\pi^2\hbar^2}{2ma^2} \leq U_0 \Rightarrow n \leq \sqrt{\frac{2ma^2U_0}{\pi^2\hbar^2}}$

b) for electron in well with $U_0 = 10\text{ eV}$, $a = 0.3\text{ nm}$,

$$n \leq \sqrt{\frac{2mc^2a^2U_0}{\pi^2(\hbar c)^2}} = \sqrt{\frac{2(511000\text{ eV})(0.3\text{ nm})^2(10\text{ eV})}{\pi^2(197.3\text{ eV·nm})^2}}$$

$$= 1.5$$

so probably only $n = 1$ is bound