6. T&D 7.50 SHO ground state $n=0$ $\psi_0(x) = A_0 e^{-x^2/2b^2}$
find A_0 by normalizing: $\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$

$$\Rightarrow A_0^2 \int_{-\infty}^{\infty} (e^{-x^2/2b^2})^2 dx = A_0^2 \int_{-\infty}^{\infty} e^{-x^2/b^2} dx = 1$$

hint: $\int_0^{\infty} e^{-\lambda x^2} dx = I_0 = \sqrt{\frac{\pi}{4\lambda}}$; since e^{-x^2/b^2} is even, $\int_{-\infty}^{0} e^{-\lambda x^2} dx = I_0$
$\quad + \lambda = 1/b^2$

$$\text{so } \int_{-\infty}^{\infty} e^{-x^2/b^2} dx = 2I_0 = 2\sqrt{\frac{\pi}{4(1/b^2)}} = 2\sqrt{\frac{\pi b^2}{4}} = \sqrt{b^2\pi}$$

$$\Rightarrow A_0^2 \sqrt{b^2\pi} = 1 \Rightarrow A_0 = (b^2\pi)^{-1/4}$$

7. T&D 7.53 SHO ground state $\psi_0(x) = (b^2\pi)^{-1/4} e^{-x^2/2b^2}$

a) classical turning points for $E = E_0 = \frac{1}{2}\hbar\omega_c$

at turning point, $\frac{1}{2}kx^2 = E \Rightarrow x = \sqrt{\frac{2E}{k}} = \sqrt{\frac{\hbar\omega_c}{k}}$

but $\omega_c = \sqrt{\frac{k}{m}} \Rightarrow k = m\omega_c^2 \Rightarrow x = \sqrt{\frac{\hbar\omega_c}{m\omega_c^2}} = \sqrt{\frac{\hbar}{m\omega_c}} = b$

b) $P(-b<x<b) = \int_{-b}^{b} |\psi_0|^2 dx = (b^2\pi)^{-1/2} \int_{-b}^{b} e^{-x^2/b^2} dx$

want to use $\int_{-1}^{1} e^{-y^2} dy = 1.49$ $\Rightarrow y = x/b$, $dy = dx/b$; $x=b \Rightarrow y=1$

$$\Rightarrow P = (b^2\pi)^{-1/2} \int_{-1}^{1} e^{-y^2} (b\, dy) = \frac{b}{\sqrt{b^2\pi}} \int_{-1}^{1} e^{-y^2} dy = \frac{1.49}{\sqrt{\pi}} = 0.84$$

= probability of being inside classical turning points

c) probability of being outside classical turning points $= 1 - 0.84 = 0.16$