(7.67), cont.

equation we need to solve is

$$\sqrt{U_0 - E} = \sqrt{E}\tan\left(\sqrt{\frac{2mE}{\hbar^2}}\frac{a}{2}\right)$$

or $$\sqrt{U_0 - 2U_0/3} = \sqrt{2U_0/3}\tan\left(\sqrt{\frac{2m\,2\pi^2\hbar^2}{2ma^2\hbar^2}}\frac{a}{2}\right)$$

$$\sqrt{1 - 2/3} = \sqrt{2/3}\tan\left(\sqrt{2}\frac{\pi}{2}\right), \quad \text{or} \quad \sqrt{3/2 - 1} = \tan\left(\sqrt{2}\frac{\pi}{2}\right)$$

solve for z: see Mathematica

$$z = 0.526\ldots$$

so $$E_1 = (0.526\ldots)\frac{\pi^2\hbar^2}{2ma^2}$$

d) repeat for $U_0 = 30E_1(\infty)$

now

$$\sqrt{U_0 - 2U_0/30} = \sqrt{2U_0/30}\tan\left(\sqrt{2}\frac{\pi}{2}\right)$$

$$\Rightarrow \sqrt{30/2 - 1} = \tan\left(\sqrt{2}\frac{\pi}{2}\right)$$

$$z = 0.802\ldots$$

closer to $E_1(\infty)$, as expected

$$\sqrt{3E_1(\infty) - 2E_1(\infty)} = \sqrt{2E_1(\infty)}\tan\left(\sqrt{\frac{2\pi^2}{a^2}}\frac{a}{2}\right)$$

$$\sqrt{3 - 2} = \sqrt{2}\tan\left(\sqrt{2}\frac{\pi}{2}\right)$$