4, (8.4) Schrödinger eqn for free particle ($U=0$) in 3D

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}=-\frac{2ME}{\hbar^2}\psi$$

want to show

$\psi = e^{+i\vec{k}\cdot\vec{r}} = e^{+i(k_x x + k_y y + k_z z)}$ satisfies S eqn

$$\frac{\partial^2\psi}{\partial x^2}=(+ik_x)^2 e^{i(k_x x+k_y y+k_z z)} = -k_x^2 e^{i\vec{k}\cdot\vec{r}} = -k_x^2\psi$$

similarly, $\frac{\partial^2\psi}{\partial y^2} = -k_y^2\psi$ & $\frac{\partial^2\psi}{\partial z^2}=-k_z^2\psi$

substitute in: $-k_x^2\psi - k_y^2\psi - k_z^2\psi = -\frac{2ME}{\hbar^2}\psi$

$$\Rightarrow \quad -(k_x^2+k_y^2+k_z^2)\psi = -\frac{2ME}{\hbar^2}\psi$$

so if $E=\frac{\hbar^2}{2M}(k_x^2+k_y^2+k_z^2)=\frac{\hbar^2k^2}{2M}$ this is an equality

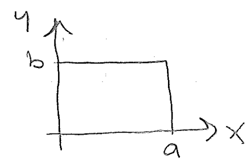
interpretation of $\vec{k}$: for free particle, $E=K=\frac{p^2}{2M}$

so $\hbar\vec{k}=\vec{p}$ or $\vec{k}=\vec{p}/\hbar$

and since (de Broglie) $p=h/\lambda$, $k=\frac{h}{\lambda\hbar}=\frac{2\pi}{\lambda}$ = wavenumber

5. (8.9) particle of mass M in 2D rigid box with sides $a+b$

$U=0$ for $0\le x\le a$ + $0\le y\le b$; $U=\infty$ otherwise



so $\psi(x,y)=0$ outside box

inside box, $\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=-\frac{2ME}{\hbar^2}\psi$

using separation of variables, write $\psi(x,y)=X(x)Y(y)$

$\Rightarrow$ S eqn inside box becomes $X''Y+XY''=-\frac{2ME}{\hbar^2}XY$

$\Rightarrow \frac{X''}{X}+\frac{Y''}{Y}=-\frac{2ME}{\hbar^2}$; separates into $\frac{X''}{X}=-k_x^2$, $\frac{Y''}{Y}=-k_y^2$; $k_x^2+k_y^2=\frac{2ME}{\hbar^2}$

so $X=B_1\sin k_x x + B_2\cos k_x x$, $Y=C_1\sin k_y y + C_2\cos k_y y$

boundary conditions:

at $x=0$, $X(0)=B_2\cos k_x x=0 \Rightarrow B_2=0$

at $y=0$, $Y(0)=C_2\cos k_y y=0 \Rightarrow C_2=0$

at $x=a$, $X(a)=B_1\sin k_x a=0 \Rightarrow k_x a=n_x\pi \Rightarrow k_x=\frac{n_x\pi}{a}$ $n_x=1,2,3,\ldots$

at $y=b$, $Y(b)=C_1\sin k_y b=0 \Rightarrow k_y b=n_y\pi \Rightarrow k_y=\frac{n_y\pi}{b}$ $n_y=1,2,3,\ldots$

so

$$\psi(x,y)=\overbrace{B_1C_1}^{A}\sin\left(\frac{n_x\pi x}{a}\right)\sin\left(\frac{n_y\pi y}{b}\right)$$

and $k_x^2+k_y^2=\frac{2ME}{\hbar^2} \Rightarrow \left(\frac{n_x\pi}{a}\right)^2+\left(\frac{n_y\pi}{b}\right)^2=\frac{2ME}{\hbar^2} \Rightarrow E=\frac{\hbar^2\pi^2}{2M}\left(\frac{n_x^2}{a^2}+\frac{n_y^2}{b^2}\right)$