3. Construct an npda that accepts the language generated by the grammar

\[ S \rightarrow aSbb \mid aab \]

Note the language is \( L = \{a^{n+2}b^{2n+1} \mid n \geq 0\} \). First convert to GNF:

\[ S \rightarrow aSBB \mid aAB; A \rightarrow a; B \rightarrow b \]

5. Construct an npda corresponding to the grammar

\[ S \rightarrow aABB \mid aAA; A \rightarrow aBB \mid a; B \rightarrow bBB \mid A \]

To convert to GNF remove the unit production

\[ S \rightarrow aABB \mid aAA; A \rightarrow aBB \mid a; B \rightarrow bBB \mid aBB \mid a \]
10. Find an npda with two states that accepts \( L = \{ a^{n}b^{2n} \mid n \geq 1 \} \)

Start with \( S \rightarrow aSbb \mid abb \) and convert to GNF: \( S \rightarrow aSBB \mid aBB; B \rightarrow b \). Derive the canonical three state npda then eliminate the \( q_1 \) state by using a special stack symbol, \( Y \), to mark it. (Note if we did not replace \( Z \) with \( Y \) the npda will accept strings of the form aabbab since after processing the second “b”, the npda is essentially back to a start state.)

Note: Can only process an “a” if \( S \) is top of stack; processing a “b” removes the \( S \) revealing the \( B \) underneath which is popped by processing a “b”.

11. Show the ndpa in Example 7.8 accepts \( L(aa^*b) \)

The transition \((q0, a, Z, AZ, q0)\) processes an initial “a”. The transition \((q0, a, A, A, q0)\) which optionally follows (since \( A \) is on the stack) will process zero to \( n \) a’s that follow. To only way to get to the final state is to process a “b” via \((q0, b, A, \lambda, q1)\) which brings the \( Z \) to the top of the stack which allows the \( \lambda \) transition to the final state. Thus the language is \( L(aa^*b) \)
Derivation of \textbf{aaab} from CF grammar on page 193

(q0, z, q2) \rightarrow a \ (q0, A, q3) \ (q3, z, q2) \rightarrow a\ a \ (q0, A, q3) \ (q3, z, q2) \rightarrow 
\hspace{1cm} a\ a\ a \ (q3, z, q2) \rightarrow a\ a\ a \ (q0, A, q1) \ (q1, z, q2) \rightarrow a\ a\ a\ b 

16. Show for every npda there exists an equivalent one satisfying conditions 1 and 2 in the preamble of Theorem 7.1 In particular explain how you would

1. empty the stack at the end and
2. handle transitions of the form \((q_i, a, A, B, q_j)\) and \((q_i, a, A, BCDx, g_j)\) where \(x \in V^*\)

1. To empty the stack create a new final state \(q_F\), add the transition \((q_f, \lambda, z, \lambda, q_F)\) and when in the old no longer final state \(q_f\) and transition pop items off the state.

2. Replace \((q_i, a, A, B, q_j)\) with the pair

\((q_i, a, A, BB, q_{ij})\) and \((q_{ij}, \lambda, B, \lambda, q_j)\) where state \(q_{ij}\) is a unique state

Replace \((q_i, a, A, BCD, g_j)\) with the pair

\((q_i, a, A, CD, q_{ij})\) and \((q_{ij}, \lambda, C, BC, q_i)\) where state \(q_{ij}\) is a unique state

An obvious cascade of similar replacements can replace \((q_i, a, A, BCDx, g_j)\) for \(x \in V^+\).
3. Construct an npda that accepts the language generated by the grammar

\[ S \rightarrow aSbb | aab \]

Note the language is \( L = \{ a^{n+2}b^{2n+1} \mid n \geq 0 \} \). First convert to GNF:

\[ S \rightarrow aSBB | aAB; A \rightarrow a; B \rightarrow b \]

5. Construct an npda corresponding to the grammar

\[ S \rightarrow aABB | aAA ; A \rightarrow aBB | a; B \rightarrow bBB | A \]

To convert to GNF remove the unit production

\[ S \rightarrow aABB | aAA; A \rightarrow aBB | a; B \rightarrow bBB | aBB | a \]
10. Find an npda with two states that accepts $L = \{a^n b^{2n} \mid n \geq 1\}$

Start with $S \rightarrow aSbb \mid abb$ and convert to GNF: $S \rightarrow aSB \mid aBB; B \rightarrow b$. Derive the canonical three state npda then eliminate the $q_1$ state by using a special stack symbol, $Y$, to mark it. (Note if we did not replace $Z$ with $Y$ the npda will accept strings of the form $aabbab$ since after processing the second “$b$”, the npda is essentially back to a start state.)

$$
\begin{align*}
S & \rightarrow aSBB \mid aBB; \\
B & \rightarrow b.
\end{align*}
$$

Note: Can only process an “$a$” if $S$ is top of stack; processing a “$b$” removes the $S$ revealing the $B$ underneath which is popped by processing a “$b$”.

11. Show the ndpa in Example 7.8 accepts $L(aa^*b)$

The transition $(q0, a, Z, AZ, q0)$ processes an initial “$a$”. The transition $(q0, a, A, A, q0)$ which optionally follows (since $A$ is on the stack) will process zero to $n$ a’s that follow. To only way to get to the final state is to process a “$b$” via $(q0, b, A, \lambda, q1)$ which brings the $Z$ to the top of the stack which allows the $\lambda$ transition to the final state. Thus the language is $L(aa^*b)$
Derivation of \texttt{aaab} from CF grammar on page 193

\[(q_0, z, q_2) \rightarrow a \quad (q_0, A, q_3) \quad (q_3, z, q_2) \rightarrow a \ a \quad (q_0, A, q_3) \quad (q_3, z, q_2) \rightarrow a \ a \ a \quad (q_3, z, q_2) \rightarrow a \ a \ a \]

16. Show for every npda there exists an equivalent one satisfying conditions 1 and 2 in the preamble of Theorem 7.1 In particular explain how you would

1. empty the stack at the end and
2. handle transitions of the form \((q_i, a, A, B, q_j)\) and \((q_i, a, A, BCDx, g_j)\) where \(x \in V^*\)

1. To empty the stack create a new final state \(q_f\), add the transition \((q_f, \lambda, z, \lambda, q_f)\) and when in the old no longer final state \(q_i\) and transition pop items off the state.

2. Replace \((q_i, a, A, B, q_j)\) with the pair

\[(q_i, a, A, BB, q_{ij})\] and \((q_{ij}, \lambda, B, \lambda, q_j)\) where state \(q_{ij}\) is a unique state

Replace \((q_i, a, A, BCD, g_j)\) with the pair

\[(q_i, a, A, CD, q_{ij})\] and \((q_{ij}, \lambda, C, BC, q_i)\) where state \(q_{ij}\) is a unique state

An obvious cascade of similar replacements can replace \((q_i, a, A, BCDx, g_j)\) for \(x \in V^+\).