**Theorem:** The base angles of an isosceles trapezoid are equal

**Introduction:** A famous theorem of geometry proves the result that the base angles of an isosceles triangle are equal. A similar result holds for an isosceles trapezoid; that is in a trapezoid where the lengths of the two non-parallel sides are equal (and the parallel sides are of different length) the base angles are equal.

Given an isosceles trapezoid ABCD where AD and BC are parallel and AB equals CD, the base angles BAD and CDA are equal. Assume BC is the shorter of the two parallel lines.

![Diagram of isosceles trapezoid with labeled angles](image)

**Proof:** Begin by constructing a line BE parallel to CD. Construct the line CE.

Because diagonal CE intersects parallel lines BC and AD angle BCE (angle 1) equals angle CED (angle 2).

Likewise because CE intersects parallel lines BE and CD angle ECD (angle 3) equals angle CEB (angle 4).

Therefore since CE is common to both triangles \( \triangle BEC \) and \( \triangle DCE \), triangles \( \triangle BEC \) and \( \triangle DCE \) are congruent by ASA.

Therefore BE equals CD and since CD equals AB, \( \triangle ABE \) is isosceles so angle BAE (angle 5) equals angle BEA (angle 6)

Since AD intersects parallel lines BE and CD angle BEA (angle 6) equals angle CDA (angle 7).

Since angle BAD (angle 5) also equals angle BEA (angle 6), angle BAD (angle 5) equals angle CDA (angle 7) and the result follows.

Therefore the base angles of an isosceles trapezoid are equal. **QED.**

**Corollary:** Angle ABC equals angle DCB.
The purpose in writing “good mathematics” is to clearly and effectively communicate a mathematical argument. I generally look for 4 things: clarity, correctness, conciseness, completeness.

Clarity – This is the “biggie”: obviously any argument presented should be clear and easy to understand. This is not always easy to do since mathematics can be difficult. Making a difficult argument clear often presents a challenge; imagination is required. Presenting a good example or examining a simple case often helps.

Correctness – obviously the logic of the argument can have no holes or flaws; this is the deal-breaker!

Conciseness – Wordiness of argument dulls the reader. An argument presented in 50 words is easier to understand than an argument in 500 words

Completeness – Did you leave anything out? Did you tie off all the loose ends? Did you forget something? Leave out an important statement necessary to make the whole argument work? Completeness is the counter-balance to conciseness and effective writing has the right mix of both.

A good argument should flow naturally – inexorably – to its conclusion.

The details of how you accomplish the above vary according to the assignment; there is no universal template when presenting a mathematical argument (or proof). However I have found that a good mathematical argument has four parts:

an introduction/background which introduces the reader to what is to come,

a statement (proposition) of what is to be proved or shown or argued,

the “proof” itself (the longest part of the write up) where all the work is done and finally

a short summary/conclusion which brings the whole write up to closure.

As an example the above is a (rather over done but effective) proof that the base angles of an isosceles triangle are equal. Not all proofs can or need to be done this way but it illustrates one way to structure a mathematical proof that is clear, correct, concise and complete.